ANALYTICAL MODELLING OF THE LINEAR WEAR OF THE INSIDE OF A PIPE TRANSPORTING LOOSE MATERIALS

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Abstract

This paper presents the results of an analysis of the linear wear of the inside of a pipe transporting loose materials. The aim is to present a mathematical model simulating the abrasive wear of individual elements of a transport pipe, depending on the volume of transported material. The model presented in this paper was developed based on the measurement data obtained from a dismantled transport system used in a railway handling terminal.

Keywords: loose materials, linear wear, analytical model, grain flow

ANALITYCZNE MODELOWANIE ŚCIERALNOŚCI WNĘTRZA RURY TRANSPORTUJĄCEJ MATERIAŁY SYPKIE

Streszczenie

W artykule przedstawiono wyniki analizy zużycia liniowego materiału wewnątrz rury służącej do transportu materiałów sypkich. Celem artykułu jest zaprezentowanie modelu matematycznego, który pozwala symulować zużycie poszczególnych elementów rury transportowej w zależności od ilości przesypanego materiału. Prezentowany w artykule model opracowany został w oparciu o dane pomiarowe uzyskane ze zdemontowanego systemu transportowego wykorzystywanego w kolejowym terminalu przeładunkowym.

Słowa kluczowe: materiały sypkie, zużycie, zużycie liniowe, model analityczny, przepływ ziarna

1. INTRODUCTION

Fast wear of transporting elements in silos and handling terminals poses a real problem. Many aspects of grain flow in such transport systems have been considered in the literature. Bevereloo [3], Brown [4] and Chang [7, 8] were among the earliest pioneers in the field. Over the years many authors have considered fluctuations, wall friction and mechanical wear [9, 10, 22, 25]. Recent years also show that the mentioned problems are still relevant and give inspiration to the construction of analytical models of transport processes in silos systems [5, 6, 14, 15, 18, 19, 20]. What is more, the transport of wet grain, causing corrosive and mechanical wear, brings an even greater problem [13, 17, 23, 24, 27]. Such conditions of transporting materials necessitate relatively frequent replacements of damaged elements, which translates into higher operating costs and causes stoppages. With computers being so ubiquitous these days, the transport of loose material is usually simulated with the use of discrete methods: specifically developed software simulates the movement of each of the thousands of transported particles. The prevalence of this approach is also proved by the substantial number of IT tools supporting it and the ever-increasing number of publications dealing with this

issue. The IT tools and mathematical models are mostly based on the Discrete Element Method (DEM). [11, 28] describe how to use and apply Yade, an open-source framework focused on DEM [11, 28]. A lot of attention is also paid to the creation of an effective model simulating the movement of loose material particles [1, 12, 16, 21]. The authors of [29, 30], who performed an accurate analysis of the usefulness of various variants of the Discrete Element Method, give a precise description of DEM's development over the years. Special approach is used to simulate the movement of wet particles [2, 26]. In this paper the focus is intentionally shifted from the discrete model (DEM) to the analytical one, as only the latter makes it possible to effectively simulate the movement of a vast number of transported particles.

2. THEORETICAL MODEL OF LINEAR WEAR OF THE INSIDE OF A TRANSPORT PIPE

The proposed model assumes that loose material (corn, rape) is transported through a circular pipe angled at $\alpha = \pi / 6 = 30º$ to the horizontal (as shown in Figure 1 and 2).

Moreover, it is assumed that the angle of slip of the transported material is β. It must be stressed that

both corn and rape have different angles of slip, depending on variety. However, the differences are minor and the vast majority of grain materials have the angle of slip equal to 30º ± 2º. Therefore, our numerical analysis assumes an average angle of slip equal to $\beta = \pi / 6 = 30º$.
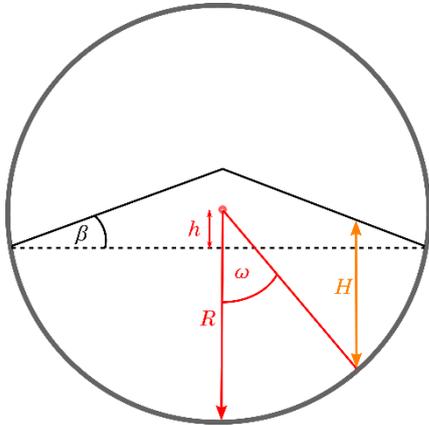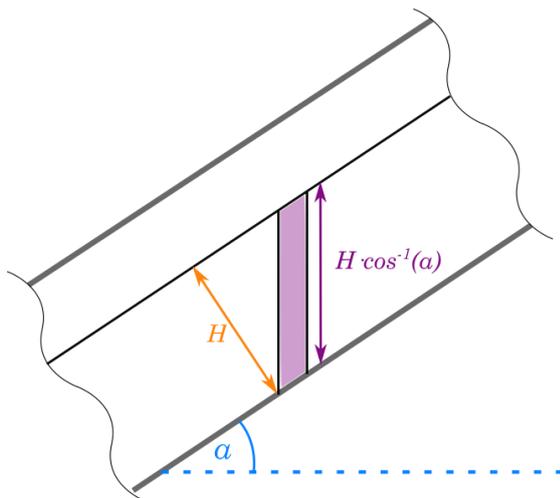

Fig. 1. Cross-section of a transport pipe of radius R.

Fig. 2. Longitudinal section of a transport pipe angled at α to the horizontal

Moreover, we assumed that the linear wear of the material inside a transport pipe is proportional to the pressure exerted by the loose material moving inside it. Let us assume that there are two sources of this pressure:
1. pressure exerted by the movement of loose material (pressure independent of the angle ω),
2. pressure exerted by the weight of loose material (pressure dependent of the angle ω).

This assumption underpins the proposed model and was made based on observation and statistical analysis of the measurement data. Later on in the paper we will demonstrate that this approach is supported and justified by a positive verification of the hypothesis on the assumptions made by us. In addition, we assume that the pipe is of radius R and that the distance between the base of the pile built

in the pipe and the geometrical axis of symmetry of the pipe equals to h (as shown in Figure 1). Based on these assumptions we develop the function of pressure:

$$p(\omega) = p_0 + p_1 f(\omega) \tag{1}$$

where p_0 is the constant pressure dependent on the speed at which the material is fed, and a p_1 is the pressure put by the weight of the loose material.

Let us assume that the height of a narrow column of the loose material is $H \cdot \cos^{-1}\alpha$ (see Figure 2) and that its cross-section area is ΔS (see Figure 3). Let $Q = \rho g \, \Delta S \, H \cos^{-1}\alpha$ be the weight of the column (ρ – density of the material, g – gravitational acceleration, ΔS – cross-section area of the column of loose material shown in Figure 3, $H \cos^{-1}\alpha$ – height of the column of loose material shown in Figure 2) By trigonometric relationships we calculate the pressure exerted by the column on the wall of the transport pipe:
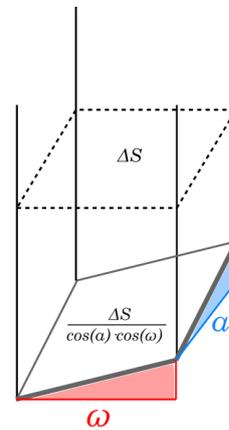

Fig. 3. Pictorial diagram of a column of loose material of with a cross-section area ΔS putting pressure on the transport pipe

$$p_1 f(\omega) = \frac{Q \cos \alpha \cos \omega}{\Delta S \cos^{-1} \alpha \cos^{-1} \omega} = \tag{2}$$

$$\rho g H \cos \alpha \cos^2 \omega.$$

As shown in Figure 1, the height H depends on the angle ω. Using simple trigonometric relationships, we obtain the following formula:

$$H(\omega) = (R \cos \omega - h) + \\ \tan \beta (\sqrt{R^2 - h^2} - R \sin | \omega |). \tag{3}$$

Finally, we obtain:

$$f(\omega) = \cos^2 \omega ((R \cos \omega - h) + \\ \tan \beta (\sqrt{R^2 - h^2} - R \sin | \omega |)), \tag{4}$$

which for $h = 0$ can be simplified to:

$$f_0(\omega) = \cos^2 \omega (R \cos \omega - \\ R \tan \beta \sin | \omega | + R \tan \beta). \tag{5}$$

Figure 4 shows the graph of function $f_0(\omega)$ for $R = 1$ and $\tan \beta = 0.577$. Please note that the unit of measure R was intentionally omitted so that the graph illustrates only the shape of the function $f_0(\omega)$.
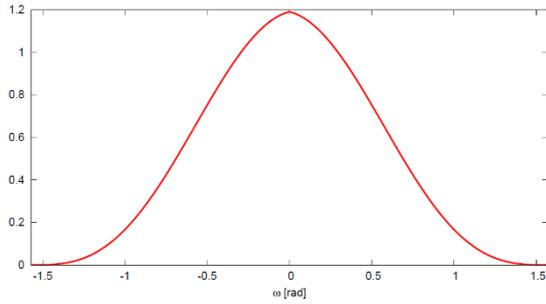
Fig. 4. Graph of function $f_0(\omega)$ for $R = 1$ and $\tan \beta = 0.577$.

At this point we are ready to define the function of linear wear of the material making up the transport pipe $U(\omega,m)$ [mm], where m is the weight of the transported loose material. Since $U \sim p$ (function U is proportional to pressure p [Pa]) and $U \sim m$ (function U is proportional to the volume of transported material m [kg]), our linear wear function is given by:

$$U(\omega,m) = mU_0 + mU_1 \cos^2 \omega((R\cos\omega - h) + \tan\beta(\sqrt{R^2 - h^2} - R\sin|\omega|)). \quad (6)$$

This formula can be simplified, if we assume that ω_0 is the maximum angle at which the transported material adheres to the inside of the pipe. If that is the case, $h = R \cdot \cos\omega_0$ and the linear wear function formula can be simplified to:

$$U(\omega,m) = m(U_0 + U_1 R \cos^2 \omega(\cos\omega - \cos\omega_0 + \tan\beta(\sin\omega_0 - \sin|\omega|))). \quad (7)$$

At this point, let us stress that U_0 and U_1 are in fact the functions of α, i.e. the angle of slip of the transport pipe. In this particular case α is assumed to be 30º and the calculations have been made based on this assumption. Making U_0 and U_1 independent of the angle of slip α is very simple only in the case of U_1. Formula (2) shows that $U_1 = K_1 \cdot \cos(\alpha)$, where K_1 is a constant depending only on the type of material. Devising a formula for U_0, however, requires further research and has not been presented in this paper.

3. ANALYSIS OF THE PROPOSED MODEL AND STATISTICAL VERIFICATION OF ITS CORRECTNESS BASED ON A DISMANTLED TRANSPORT PIPE

The model presented in the previous section defines the linear wear function $U(\omega,m)$, which determines the reduction of the thickness of the pipe during transport. For the purpose of statistical analysis, the linear wear of the material (steel C30) inside a transport pipe dismantled from a handling terminal was measured. The measurements were made for 5 cross-sections and 14 values of the angle ω equal to 0º, 4.2º, 8.4º, 12.5º, 16.7º, 20.9º, 25.1º, 29.2º, 33.4º, 37.6º, 41.8º, 45.9º, 50.1º and 54.3º in each cross-section. Figure 5 shows the

points in which the thickness of the walls of the transport pipe was measured.
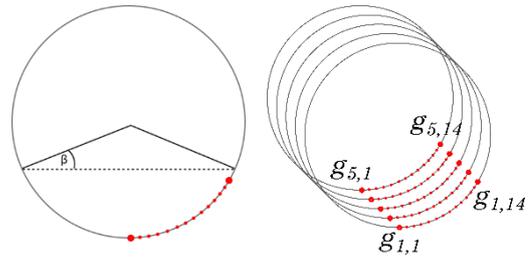

Fig. 5. Measure points of the thickness of a transport pipe.

No measurements were performed for negative angles due to a joint located symmetrically on the other side, which grossly distorted the measurement results. Originally the wall of the transport pipe was 5.3 [mm] thick. After transporting 120 thousand tons of grain, the thickness measured in five points marked in Figure 5 was reduced to the levels shown in Table 1.

Table 1

Angle [º]	Thickness of the walls of the transport pipe for each measurement [mm]				
	$g_{1,*}$	$g_{2,*}$	$g_{3,*}$	$g_{4,*}$	$g_{5,*}$
0.0	4.16	4.12	4.12	4.17	4.18
4.2	4.24	4.24	4.22	4.25	4.20
8.4	4.28	4.32	4.29	4.30	4.27
12.5	4.43	4.40	4.46	4.44	4.42
16.7	4.55	4.51	4.49	4.51	4.52
20.9	4.62	4.69	4.65	4.60	4.62
25.1	4.76	4.78	4.75	4.74	4.80
29.2	4.84	4.82	4.83	4.86	4.85
33.4	4.94	4.94	4.96	4.98	4.95
37.6	5.02	5.04	5.07	5.00	5.05
41.8	5.07	5.13	5.12	5.14	5.10
45.9	5.09	5.12	5.12	5.11	5.12
50.1	5.18	5.15	5.12	5.14	5.14
54.3	5.14	5.17	5.19	5.20	5.12

Given that originally the wall was 5.3 [mm] thick, the linear wear ($u_{i,j} = 5.3 - g_{i,j}$) of the material is shown in Table 2.

Table 2

Angle [º]	Linear wear of the walls of the transport pipe for each measurement [mm]				
	$u_{1,*}$	$u_{2,*}$	$u_{3,*}$	$u_{4,*}$	$u_{5,*}$
0.0	1.14	1.18	1.18	1.13	1.12
4.2	1.06	1.06	1.08	1.05	1.10
8.4	1.02	0.98	1.01	1.00	1.03
12.5	0.87	0.90	0.84	0.86	0.88
16.7	0.75	0.79	0.81	0.79	0.78
20.9	0.68	0.61	0.65	0.70	0.68
25.1	0.54	0.52	0.55	0.56	0.50
29.2	0.46	0.48	0.47	0.44	0.45
33.4	0.36	0.36	0.34	0.32	0.35
37.6	0.28	0.26	0.23	0.30	0.25
41.8	0.23	0.17	0.18	0.16	0.20
45.9	0.21	0.18	0.18	0.19	0.18
50.1	0.12	0.15	0.18	0.16	0.16
54.3	0.16	0.13	0.11	0.10	0.18

By calculating the average values and the measurement error for individual angles, we obtain the following relationships for the linear wear function $U(\omega,m)$:

Table 3

U function	Measure avg. [mm]	Measure error [mm]
$U(0.0º, 1.2e8)$	1.15	± 0.08
$U(4.2º, 1.2e8)$	1.07	± 0.06
$U(8.4º, 1.2e8)$	1.01	± 0.06
$U(12.5º, 1.2e8)$	0.87	± 0.07
$U(16.7º, 1.2e8)$	0.78	± 0.07
$U(20.9º, 1.2e8)$	0.66	± 0.11
$U(25.1º, 1.2e8)$	0.53	± 0.07
$U(29.2º, 1.2e8)$	0.46	± 0.05
$U(33.4º, 1.2e8)$	0.35	± 0.05
$U(37.6º, 1.2e8)$	0.26	± 0.08
$U(41.8º, 1.2e8)$	0.19	± 0.08
$U(45.9º, 1.2e8)$	0.19	± 0.04
$U(50.1º, 1.2e8)$	0.15	± 0.07
$U(54.3º, 1.2e8)$	0.14	± 0.10

To complete the analysis, we also need to know the radius of the transport pipe, which in this case was $R = 130$ [mm]. The performed examination also showed that the angle at which the transported material adhered to the surface of the pipe ranged approximately from -60º to 60º. Therefore, we assume that $\omega_0 = 60º$. In addition, we calculate the angle of slip for the transported grain $\beta = 30º$, for which we assume that $\tan \beta = 0.577$. Under these assumptions the formula (7) expressing the $U(\omega,m)$ is given by:

$$U(\omega, m) = m(U_0 + 130 \cdot U_1 \cos^2 \omega(\cos \omega - 0.58 \sin |\omega| - 0.005)). \quad (8)$$

Using the least squares method we determine the following coefficients of the linear wear function:

1. $U_0 = 1.52e\text{-}10$ [mm/kg],
2. $U_1 = 7.03e\text{-}11$ [mm/kg].

Please note that U_0 and U_1 were calculated for a specific material making up the transport pipe, namely C30 steel. U_0 and U_1 may differ depending on the material and should thus be determined empirically. Finally, we obtain:

$$U(\omega, m) = \frac{m}{10^{10}} (1.52 + 0.703 \cos^2 \omega(\cos \omega - 0.58 \sin |\omega| - 0.005)). \quad (9)$$

Figure 6 presents a graph of the function of linear wear of the material making up the inside of the transport pipe $U(\omega,1.2e8)$. It confronts the results obtained in the proposed model (solid red line) with the measurements made for a real system for transporting grain (blue error bars). As you can see, all measuring points of the real system conform with the theoretical model to an accuracy of the measurement error. Consequently, we can assume

that the hypothesis on the correctness of the proposed model is true.
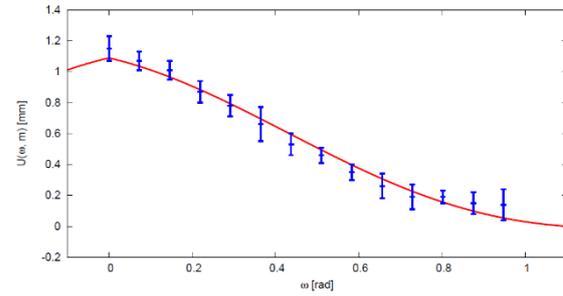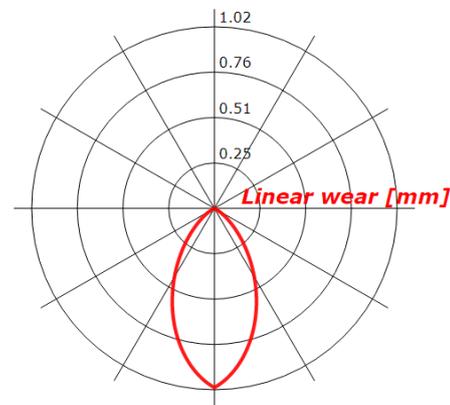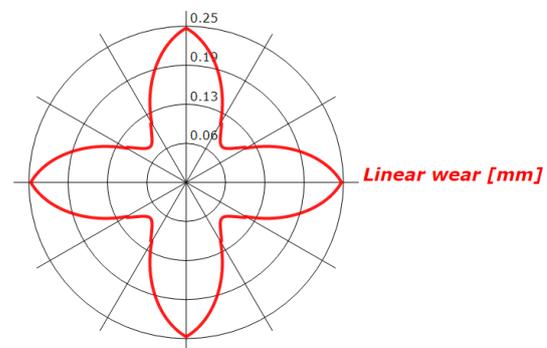

Fig. 6 Graph of the linear wear function $U(\omega,1.2e8)$.

Figure 7 presents a polar graph of the linear wear function. One can easily see that the wear of the transport pipe is extremely heterogeneous.

Fig. 7. Polar graph of the linear wear function $U(\omega,1.2e8)$ without pipe rotation.

A very natural way to make a pipe more durable is its periodic rotation. Figures 8, 9 and 10 present a polar graph of the linear wear function with pipe rotation. The reader can see that periodic rotation by the angle of 30º gives almost homogenous distribution of the linear wear.

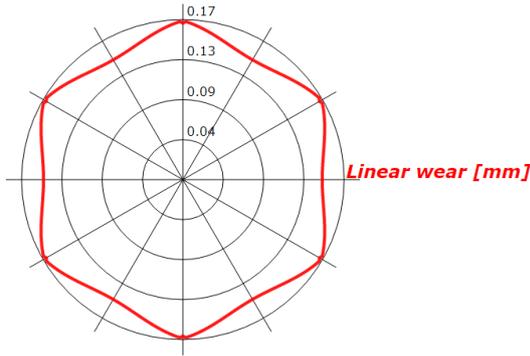Fig. 8 Polar graph of the linear wear function $U(\omega,1.2e8)$ with pipe rotation (90º every 0.3e8 [kg]).

Fig. 9 Polar graph of the linear wear function
$U(\omega,1.2e8)$ with pipe rotation (60º every 0.2e8
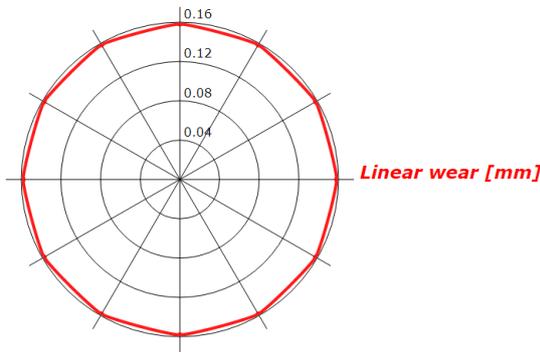[kg]).

Fig. 10 Polar graph of the linear wear function
$U(\omega,1.2e8)$ with pipe rotation (30º every 0.1e8
[kg]).

4. MODELLING SOFTWARE

To simulate the linear wear of a transport pipe we developed a Web browser application, which can be downloaded using the following link:
https://achmie.v.prz.edu.pl/materialy-do-pobrania/materialy-ogolnodostepne
The software is free of charge and consists of four files: pipe.html (main application), plot.js (JavaScript code to generate Cartesian plots), polarPlot.js (JavaScript code to generate polar plots) and uFunction.js (JavaScript code that implements the *U* linear wear function based on the model presented in this article). To run the software, simply open pipe.html in your browser, insert numeric data and click *Refresh* button to start the numerical analysis.

Before starting the simulation, it is necessary to set up the following data (like in Figure 11):
1. *Δm* [kg] - mass of the material transported through the pipe during a single cycle (without pipe rotation).
2. *M* [kg] - total mass of transported material.
3. *R* [mm] - radius of the transport pipe.
4. *δ* [º] - rotation angle of the transport pipe after a single cycle (*Δm* kilograms of material).

Then *Refresh* button can be used to simulate the linear wear process in the transport pipe.

Figure 12 shows the Cartesian plot of the linear wear. The ω angle in degrees goes on the x-axis and the y-axis represents linear wear in millimetres.
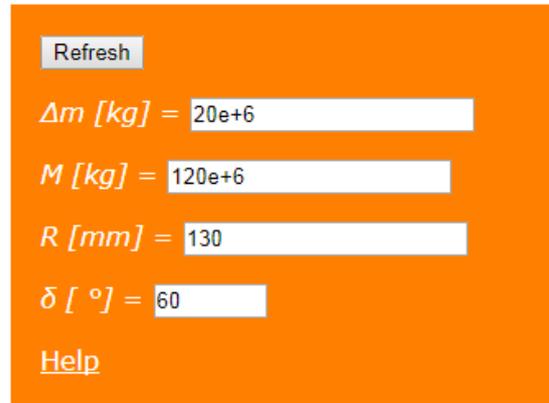

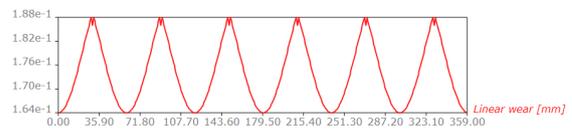Fig. 11 Application menu with fields for numeric data.

Fig. 12 Linear wear plot in Cartesian coordinate system.

Figures 13 and 14 illustrate how *Next* and *Back* buttons can be used to produce polar plots of linear wear after each cycle of pipe rotation (the transport pipe is rotated after every Δm kilograms of material).
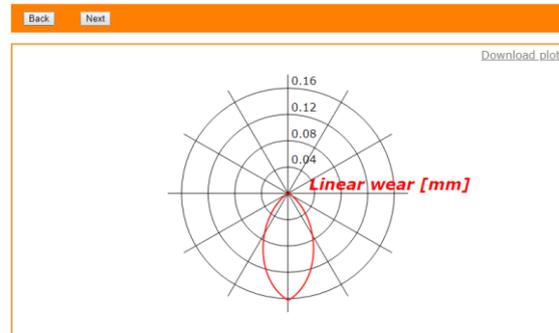

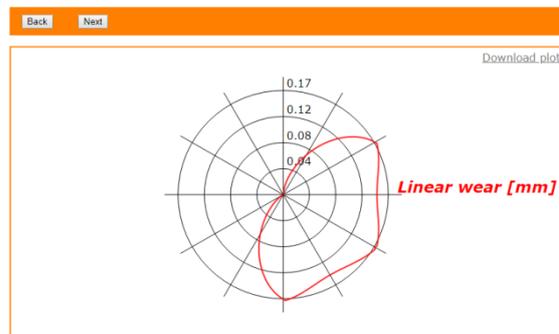Fig. 13 Linear wear polar plot after transporting the first part of material (*Δm*).

Fig. 14 Linear wear polar plot after transporting the first three parts of material (3 · *Δm*) and a 60º cyclic pipe rotation.

5. CONCLUSIONS

Both the measurements of the elements used and the theoretical model confirmed that the transport pipe wears out very unevenly. However, if we equip the pipe with a mechanism allowing it to rotate at a specific angle after transporting a given volume of loose material, its lifespan can be prolonged considerably. Although in this paper the loose material was assumed to be corn or rape, it is also possible to perform analogical calculations for other materials. However, at this point it must be stressed that various materials may require rotating the transport pipe at a different angle. The same applies to the value h, which depends strongly on the speed at which the material is fed into the transport pipe. In other words, the type of transported material and the fill rate of the pipe are the key factors determining the angle for which the linear wear of the pipe is relatively homogenous.

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