

STATISTICAL IDENTIFICATION OF COMPLEX TECHNICAL SYSTEM OPERATION PROCESS

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Summary

The paper is concerned with the methods and procedures for identification of unknown parameters of a general probability model of a complex technical system operation process and their practical application. The general multistate model of a complex technical system operation process is proposed and the procedure of identifying its basic unknown parameters is presented. There are also suggested typical distribution functions describing the system operation process conditional sojourn times at particular operation states and the procedure of identifying their unknown parameters is proposed. An illustration of the proposed methods and procedures practical application to identifying the port oil piping transportation system operation process and its operation characteristics prediction is presented.

Keywords: operation proces; modeling; identification; prediction; port transport.

IDENTYFIKACJA STATYSTYCZNA PROCESU EKSPLOATACJI ZŁOŻONEGO SYSTEMU TECHNICZNEGO

Streszczenie

Artykuł dotyczy metod i procedur identyfikacji nieznanymi parametrów ogólnego modelu probabilistycznego procesu eksploatacji złożonego systemu technicznego oraz ich praktycznego zastosowania. Zaproponowany jest ogólny wielostanowy model procesu eksploatacji złożonego systemu technicznego oraz przedstawiona jest procedura identyfikacji jego nieznanymi podstawowych parametrów. Zaproponowane są także typowe dystrybuanty warunkowe opisujące czasy przebywania procesu eksploatacji systemu w poszczególnych stanach eksploatacyjnych oraz zaproponowana jest procedura identyfikacji ich nieznanymi parametrów. Przedstawiona jest ilustracja praktycznego zastosowania proponowanych metod i procedur do identyfikacji procesu eksploatacji portowego rurociągowego systemu transportu paliwa i do predykcji jego charakterystyk eksploatacyjnych.

Słowa kluczowe: proces eksploatacji; modelowanie; identyfikacja; predykcja; transport portowy.

1. INTRODUCTION

Modeling the complex system operation process is complicated because of the large number of the operation states, impossibility of their precise defining and exact describing the transitions between them. The changes of the operation states of the system operation process cause the changes of the system reliability structure and also the changes of its components reliability parameters. The general joint model linking the system reliability model with the model of its operation process is constructed in [1] and [2]. To apply this general model practically to the evaluation and prediction of real complex technical systems reliability it is necessary to elaborate the statistical methods concerned with determining the unknown parameters of the proposed model. Particularly, concerning the system

operation process, the methods of estimating the probabilities of the initials system operation states, the probabilities of transitions between the system operation states and the distributions of the sojourn times of the system operation process at the operation states should be proposed [3]. The methods of testing the hypotheses concerned with the conditional sojourn times of the system operation process at the operation states should be also elaborated.

2. MATHEMATICAL MODEL OF COMPLEX TECHNICAL SYSTEM OPERATION PROCESS

We assume that the system during its operation process is taking $\nu, \nu \in N$, different operation states z_1, z_2, \dots, z_ν . Further, we define the system

operation process $Z(t)$, $t \in \langle 0, +\infty \rangle$, with discrete operation states from the set $\{z_1, z_2, \dots, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process [1]-[9], with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l , $b, l = 1, 2, \dots, v$, $b \neq l$.

Under these assumptions, the system operation process may be described by:

- the vector $[p_b(0)]_{1 \times v}$ of the initial probabilities

$$p_b(0) = P(Z(0) = z_b), \quad b = 1, 2, \dots, v,$$

of the system operation process $Z(t)$ staying at the operation states at the moment $t = 0$;

- the matrix $[p_{bl}]_{1 \times v}$ of probabilities

$$p_{bl}, \quad b, l = 1, 2, \dots, v, \quad b \neq l,$$

of the system operation process $Z(t)$ transitions between the operation states z_b and z_l ;

- the matrix $[H_{bl}(t)]_{1 \times v}$ of conditional distribution functions

$$H_{bl}(t) = P(\theta_{bl} < t), \quad b, l = 1, 2, \dots, v, \quad b \neq l,$$

of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states.

The mean values of the conditional sojourn times θ_{bl} of the system operation process $Z(t)$ are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^{\infty} t dH_{bl}(t), \quad b, l = 1, 2, \dots, v, \quad b \neq l. \quad (1)$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b , $b = 1, 2, \dots, v$, of the system operation process $Z(t)$ at the operation states z_b , $b = 1, 2, \dots, v$, are given by [1], [2]

$$H_b(t) = \sum_{l=1}^v p_{bl} H_{bl}(t), \quad b = 1, 2, \dots, v. \quad (2)$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b , $b = 1, 2, \dots, v$, at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^v p_{bl} M_{bl}, \quad b = 1, 2, \dots, v, \quad (3)$$

where M_{bl} are defined by the formula (1).

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b), \quad t \in \langle 0, +\infty \rangle, \quad b = 1, 2, \dots, v, \quad (4)$$

are given by [1]-[9]

$$p_b = \lim_{t \rightarrow \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^v \pi_l M_l}, \quad b = 1, 2, \dots, v, \quad (5)$$

where M_b , $b = 1, 2, \dots, v$, are given by (3), while the steady probabilities π_b of the vector $[\pi_b]_{1 \times v}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^v \pi_l = 1. \end{cases} \quad (6)$$

Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b , $b = 1, 2, \dots, v$, during the fixed system operation time. It is well known [2], [6] that the system operation process total sojourn times $\hat{\theta}_b$ at the particular operation states z_b , for sufficiently large operation time θ , have approximately normal distributions with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta, \quad b = 1, 2, \dots, v, \quad (7)$$

where p_b are given by (5).

3. PROCEDURE OF IDENTIFYING UNKNOWN PARAMETERS OF COMPLEX SYSTEM OPERATION PROCESS

3.1. Methodology of description of the complex technical system

The description of the complex technical systems should include at least the following items:

- the system designation,
- the system operation conditions,
- the system subsystem and components.

3.2. Methodology of defining the parameters of the system operation process

To make the estimation of the unknown parameters of the system operations process the experiment delivering the necessary statistical data should be precisely planned.

Firstly, before the experiment, we should perform the following preliminary steps [3]:

- i) to analyze the system operation process;

- ii) to fix or to define its following general parameters:
 - the number of the operation states of the system operation process ν ,
 - the operation states of the system operation process z_1, z_2, \dots, z_ν ;
- iii) to fix the possible transitions between the system operation states;
- iv) to fix the set of the unknown parameters of the system operation process semi-Markov model.

3.3. Procedure of the system operation process data collection

To estimate the unknown parameters of the system operations process, during the experiment, we should collect necessary statistical data performing the following steps [3]:

- i) to fix and to collect the following statistical data necessary to evaluating the probabilities of the initial states of the system operations process:
 - the duration time of the experiment Θ ,
 - the number $n(0)$ of the observed realizations of the system operation process,
 - the numbers

$$n_1(0), n_2(0), \dots, n_\nu(0),$$

where

$$n_1(0) + n_2(0) + \dots + n_\nu(0) = n(0),$$

of staying of the operation process respectively in the operations states z_1, z_2, \dots, z_ν , at the initial moment $t = 0$ of all $n(0)$ observed realizations of the system operation process in the form of the vector $[n(0)]_{1 \times \nu}$;

- ii) to fix and to collect the following statistical data necessary to evaluating the transient probabilities between the system operation states:
 - the numbers

$$n_{bl}, b, l = 1, 2, \dots, \nu, b \neq l,$$

of the transitions of the system operation process from the operation state z_b to the operation state z_l during all observed realizations of the system operation process in the form of the matrix $[n_{bl}]_{\nu \times \nu}$;

- the numbers

$$n_b = n_{b1} + n_{b2} + \dots + n_{b\nu}, b = 1, 2, \dots, \nu,$$

of departures of the system operation process from the operation states z_b in the form of the column $[n_b]_{\nu \times 1}$;

- iii) to fix and to collect the following statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states:
 - the realizations

$$\theta_{bl}^k, k = 1, 2, \dots, n_{bl}, b, l = 1, 2, \dots, \nu, b \neq l,$$

of the conditional sojourn times θ_{bl} of the system operations process at the operation state z_b when the next transition is to the operation state z_l during the observation time Θ .

3.4. Procedure of evaluating the unknown parameters of the system operation process

After collecting the statistical data, it is possible to estimate the unknown parameters of the system operation process performing the following steps [3]:

- i) to determine the vector $[p(0)]_{1 \times \nu}$ of the realizations of the probabilities $p_b(0)$, $b = 1, 2, \dots, \nu$, of the initial states of the system operation process, according to the formula

$$p_b(0) = \frac{n_b(0)}{n(0)} \text{ for } b = 1, 2, \dots, \nu; \tag{8}$$

- ii) to determine the matrix $[p_{bl}]_{\nu \times \nu}$ of the realizations of the probabilities p_{bl} , $b, l = 1, 2, \dots, \nu$, of the system operation process transitions from the operation state z_b to the operation state z_l , according to the formula

$$p_{bl} = \frac{n_{bl}}{n_b} \text{ for } b, l = 1, 2, \dots, \nu, b \neq l, p_{bb} = 0, \tag{9}$$

$$b = 1, 2, \dots, \nu;$$

- iii) to determine the following empirical characteristics of the realizations of the conditional sojourn time of the system operation process at the operation states:

- the realizations of the mean values $\bar{\theta}_{bl}$ of the conditional sojourn times θ_{bl} of the system operation process at the operation state z_b when the next transition is to the operation state z_l , according to the formula

$$\bar{\theta}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} \theta_{bl}^k, b, l = 1, 2, \dots, \nu, b \neq l \tag{10}$$

- the number \bar{r}_{bl} , of the disjoint intervals

$$I_j = \langle a_{bl}^j, b_{bl}^j \rangle, \quad j = 1, 2, \dots, \bar{n}_{bl},$$

that include the realizations

$$\theta_{bl}^k, \quad k = 1, 2, \dots, n_{bl},$$

of the conditional sojourn times θ_{bl} , the lengths d_{bl} and the ends a_{bl}^j, b_{bl}^j of these intervals and the numbers n_{bl}^j of the realizations θ_{bl}^k in these intervals [2].

3.5. Procedure of identifying the distributions of the system conditional sojourn times in operation states

To formulate and next to verify the hypothesis concerning the form of the distribution function $H_{bl}(t)$ of the system conditional sojourn time θ_{bl} on the basis of its realizations

$$\theta_{bl}^k, \quad k = 1, 2, \dots, n_{bl},$$

it is necessary to proceed according to the following scheme:

- to construct and to plot the realization of the histogram of the system conditional sojourn time θ_{bl} at the operation state, defined by the following formula

$$\bar{h}_{n_{bl}}(t) = \frac{n_{bl}^j}{n_{bl}} \quad \text{for } t \in I_j, \quad (11)$$

- to analyze the realization of the histogram, comparing it with the graphs of the density functions $h_{bl}(t)$ of the previously distinguished distributions, to select one of them and to formulate the null hypothesis H_0 , concerning the unknown form of the distribution function $H_{bl}(t)$ of the conditional sojourn time θ_{bl} ,
- to estimate the parameters of the selected distribution of the conditional sojourn times of the system operation process at the operation state in the way given in [2],
- to verify the hypothesis H_0 using the chi-square test [2].

3.6. Procedure of identifying the mean values of the system conditional sojourn times in operation states

After identifying the matrix $[h_{bl}(t)]_{v \times v}$ of the conditional density functions of the system conditional sojourn times θ_{bl} , $b, l = 1, 2, \dots, v$, $b \neq l$,

at the operation states corresponding to the matrix $[H_{bl}(t)]_{v \times v}$ of distribution functions, it is possible to determine the mean values of the system conditional sojourn times at the operation states either using (1) or the direct formulae for the distinguished distributions fixed in [2]. In the case when the identification of the conditional density functions of the system conditional sojourn times θ_{bl} , $b, l = 1, 2, \dots, v$, $b \neq l$, at the operation states is not possible we may determine the approximate empirical values of the system conditional sojourn times in the operation states according to the formula (10) or use their approximate values coming from experts.

4. STATISTICAL IDENTIFICATION OF PORT OIL PIPING TRANSPORTATION SYSTEM OPERATION PROCESS

4.1. Description of port oil piping transportation system operation process

The considered port oil piping transportation system is the main part of the Oil Terminal in Dębogórze that is designated for the reception from ships, the storage and sending by carriages or by cars the oil products such like petrol and oil. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products. The considered terminal is composed of three parts *A*, *B* and *C*, linked by the piping transportation systems with the pier. The scheme of this terminal is presented in Figure 1. The unloading of tankers is performed at the pier placed in the Port of Gdynia. The pier is connected with terminal part *A* through the transportation subsystem S_1 built of two piping lines composed of steel pipe segments with diameter of 600 mm. In the part *A* there is a supporting station fortifying tankers pumps and making possible further transport of oil by the subsystem S_2 to the terminal part *B*. The subsystem S_2 is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part *B* is connected with the terminal part *C* by the subsystem S_3 . The subsystem S_3 is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part *C* is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the Port of Gdynia and further to the interior of the country.

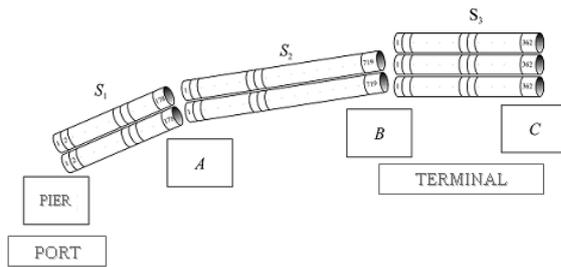


Fig. 1. The scheme of the port oil piping transportation system

The oil pipeline system consists three subsystems:

- the subsystem S_1 composed of two identical pipelines, each composed of 178 pipe segments of length 12m and two valves,
- the subsystem S_2 composed of two identical pipelines, each composed of 717 pipe segments of length 12m and two valves,
- the subsystem S_3 composed of three different pipelines, each composed of 360 pipe segments of either 10 m or 7,5 m length and two valves.

4.2. Defining the parameters of the port oil piping transportation system operation process

Taking into account the expert opinion on the operation process of the considered port oil pipeline transportation system we fix [10]:

- the number of the pipeline system operation process states $\nu = 7$ and we distinguish the following as its seven operation states:
 - an operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3 ,
 - an operation state z_2 – transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines of S_3 ,
 - an operation state z_3 – transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines of the subsystem S_2 and one out of two pipelines of the subsystem S_1 ,
 - an operation state z_4 – transport of two kinds of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1 , one out of two pipelines of the subsystem S_2 and two out of three pipelines of the subsystem S_3 ,
 - an operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 ,

- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3 , and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 ,
- an operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3 , and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3 .

Moreover, we fix that there are possible the transitions between all system operation states. Thus, the unknown parameters of the system operation process semi-Markov model are:

- the initial probabilities $p_b(0)$, $b = 1, 2, \dots, 7$, $b \neq l$, of the pipeline system operation process transients in the particular states z_b at the moment $t = 0$,
- the transition probabilities p_{bl} , $b, l = 1, 2, \dots, 7$, of the pipeline system operation process from the operation state z_b into the operation state z_l ,
- the distributions of the conditional sojourn times θ_{bl} , $b, l = 1, 2, \dots, 7$, $b \neq l$, in the particular operation states and their mean values.

To identify all these parameters of the pipeline system operation process the collected statistical data about this process presented in [2] are needed.

4.3. The port oil piping transportation system operation process data collection

The collected statistical data necessary to evaluating the initial transient probabilities of the piping system operation process in the particular states are:

- the pipeline system operation process observation/experiment time $\Theta = 329$ days = 47 weeks,
- the number of the pipeline system operation process realizations $n(0) = 41$,
- the vector of realization $n_b(0)$ of the number of the pipeline system operation process transients in the particular operation states z_b at the initial moment $t = 0$

$$[n_b(0)] = [14, 2, 0, 0, 9, 8, 8].$$

The collected statistical data necessary to evaluating the transition probabilities of the pipeline system operation process between the operation states are:

- the matrix realization n_{bl} of the numbers of pipeline system operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 329$ days

$$[n_{bl}]_{7 \times 7} = \begin{bmatrix} 0 & 1 & 1 & 0 & 24 & 5 & 14 \\ 1 & 0 & 0 & 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 21 & 1 & 0 & 1 & 0 & 10 & 10 \\ 2 & 0 & 0 & 0 & 14 & 0 & 5 \\ 17 & 2 & 0 & 0 & 7 & 7 & 0 \end{bmatrix},$$

- the column of realization n_b of the total numbers of the pipeline system operation process transitions from the operation state z_b during the experiment time $\Theta = 329$ days

$$[n_b]_{7 \times 1} = [45, 5, 1, 1, 43, 21, 33]^T.$$

The exemplary collected statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the port oil pipeline transportation system operation process at the operation states the variable θ_{15} are as follows:

- the number of realizations $n_{15} = 24$,
- the realizations:

$$\theta_{15}^1 = 930, \theta_{15}^2 = 3840, \theta_{15}^3 = 1290, \theta_{15}^4 = 480,$$

$$\theta_{15}^5 = 5575, \theta_{15}^6 = 4680, \theta_{15}^7 = 4350, \theta_{15}^8 = 2100,$$

$$\theta_{15}^9 = 840, \theta_{15}^{10} = 2460, \theta_{15}^{11} = 1560, \theta_{15}^{12} = 1020,$$

$$\theta_{15}^{13} = 1860, \theta_{15}^{14} = 960, \theta_{15}^{15} = 930, \theta_{15}^{16} = 910,$$

$$\theta_{15}^{17} = 480, \theta_{15}^{18} = 410, \theta_{15}^{19} = 960, \theta_{15}^{20} = 480,$$

$$\theta_{15}^{21} = 1440, \theta_{15}^{22} = 4710, \theta_{15}^{23} = 540, \theta_{15}^{24} = 5180.$$

4.4. Evaluating unknown basic parameters of port oil piping transportation system operation process

On the basis of the statistical data, using the formulae given in Section III, it is possible to evaluate

- the vector of realizations

$$[p(0)]_{1 \times 7} = [0.34, 0.05, 0, 0, 0.23, 0.19, 0.19]$$

- of the initial probabilities $p_b(0)$, $b = 1, 2, \dots, 7$, (8) of the pipeline system operation process transients at the operation states z_b at the moment $t = 0$,
- the matrix of realizations

$$[p_{bl}]_{7 \times 7} = \begin{bmatrix} 0 & 0.022 & 0.022 & 0 & 0.534 & 0.111 & 0.311 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.488 & 0.023 & 0 & 0.023 & 0 & 0.233 & 0.233 \\ 0.095 & 0 & 0 & 0 & 0.667 & 0 & 0.238 \\ 0.516 & 0.064 & 0 & 0 & 0.226 & 0.194 & 0 \end{bmatrix} \quad (12)$$

- of the transition probabilities p_{bl} , $b, l = 1, 2, \dots, 7$, (9) of the pipeline system operation process between the states.

4.5. Identifying distributions of conditional sojourn times at operation states of port oil piping transportation system

On the basis of statistical data, it is possible to determine the empirical characteristics of the conditional sojourn times of the pipeline system operation process at the operation states. Particularly, for θ_{15} , we have:

- the realizations of the mean value of the conditional sojourn times θ_{15} , calculated according to (10)

$$\bar{\theta}_{15} = \frac{1}{24} \sum_{k=1}^{24} \theta_{15}^k = 1999.4 \quad b, l = 1, 2, \dots, 7 \quad b \neq l,$$

- the number $\bar{r}_{15} = 5$ of the disjoint intervals that include the realizations θ_{15}^k of the conditional sojourn times θ_{15} , the length $d_{15} = 1291$ and the ends a_{bl}^j , b_{bl}^j of these intervals:

$$a_{15}^1 = 0, \quad b_{15}^1 = a_{15}^2 = 1291, \quad b_{15}^2 = a_{15}^3 = 2582,$$

$$b_{15}^3 = a_{15}^4 = 3873, \quad b_{15}^4 = a_{15}^5 = 5164, \quad b_{15}^5 = 6455, \quad (13)$$

- the numbers n_{15}^j of the realizations θ_{15}^k in these intervals

$$n_{15}^1 = 13, \quad n_{15}^2 = 5, \quad n_{15}^3 = 1, \quad n_{15}^4 = 4, \quad n_{15}^5 = 1. \quad (14)$$

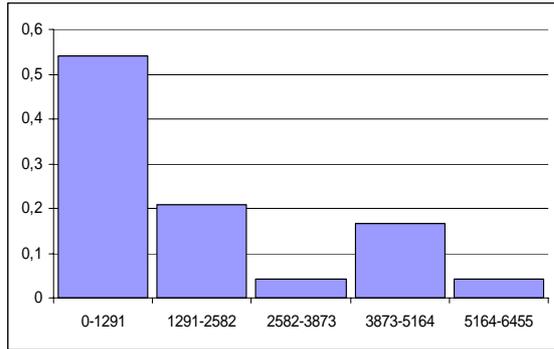


Fig. 2. Histogram of the conditional sojourn time θ_{15}

The realization of the histogram defined by (11) of the system conditional sojourn time θ_{15} constructed on the basis of the empirical results (13)-(14) is presented in Figure 2.

Using the procedure given in [2] we may verify the hypotheses on the distributions of the conditional sojourn times θ_{bl} , $b, l = 1, 2, \dots, 7, b \neq l$, at the operation states. For instance, the conditional sojourn time θ_{15} has a chimney distribution with the density function:

$$h_{15}(t) = \begin{cases} 0, & t < 0, \\ 0.0003, & 0 \leq t < 1721.67, \\ 0.00007, & 1721.67 \leq t < 6886.67, \\ 0, & t \geq 6886.67. \end{cases} \quad (15)$$

4.6. Identifying mean values of system conditional sojourn times at operation states of port oil piping transportation system

For the distributions identified in Section IV E, using the formula (1), we can find the mean values of the conditional sojourn times in the particular operation states. For instance, after applying (1) and (15), we get $M_{15} = 1999.4$. In the remaining cases, because of lack of sufficiently large sets of empirical data for testing the hypotheses, it is possible to find only the approximate values of the mean values $M_{bl} = E[\theta_{bl}]$ of the conditional sojourn times at the operation states, using (10), that are as follow:

$$\begin{aligned} M_{12} &= 1920, M_{13} = 480, M_{16} = 1250, M_{17} = 1129.6, \\ M_{21} &= 9960, M_{27} = 810, M_{31} = 575, M_{47} = 380, \\ M_{51} &= 874.7, M_{52} = 480, M_{54} = 300, M_{56} = 436.3, \\ M_{57} &= 1042.5, M_{71} = 874.1. \end{aligned}$$

This way, the port oil piping transportation system operation process is approximately identified and we may predict its main characteristics.

5. PORT OIL PIPING TRANSPORTATION SYSTEM OPERATION PROCESS PREDICTION

Applying (3), the unconditional mean sojourn times of the piping system operation process at the operation states are:

$$\begin{aligned} M_1 &\cong 1610.52, M_2 \cong 2640, M_3 = 575, M_4 = 380, \\ M_5 &\cong 789.35, M_6 \cong 475.76, M_7 \cong 1497.16. \end{aligned} \quad (16)$$

Considering (12) in the system of equations (6), we get its following solution

$$\begin{aligned} \pi_1 &\cong 0.291, \pi_2 \cong 0.027, \pi_3 \cong 0.006, \pi_4 \cong 0.007, \\ \pi_5 &\cong 0.301, \pi_6 \cong 0.144, \pi_7 \cong 0.224. \end{aligned}$$

Hence and from (16), after applying (5), it follows that the limit values of the piping system operation process transient probabilities $p_b(t)$ at the operation states z_b are given by

$$\begin{aligned} p_1 &= 0.395, p_2 = 0.060, p_3 = 0.003, p_4 = 0.002, \\ p_5 &= 0.200, p_6 = 0.058, p_7 = 0.282. \end{aligned}$$

Substituting the above transient probabilities at operation states into (7), we get the mean values of the port oil piping transportation system operation process total sojourn times at the particular operation states during $\theta = 1$ year:

$$\begin{aligned} \hat{M}_1 &\cong 144, \hat{M}_2 \cong 22, \hat{M}_3 \cong 1, \hat{M}_4 \cong 1, \hat{M}_5 \cong 73, \\ \hat{M}_6 &\cong 21, \hat{M}_7 \cong 103 \text{ days.} \end{aligned}$$

6. CONCLUSIONS

The way of the identification of the operation process of complex system including the formulae and procedures for estimating unknown parameters of the operation process semi-Markov and its characteristics prognosis is proposed. Its application to the port oil piping transportation system operation process unknown parameters estimation and operation characteristic prediction proves of the proposed formulae and procedures high practical importance.

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