A MATHEMATICAL MODEL FOR DESIGNING TOOTH SURFACES IN SPIRAL BEVEL GEARS AND GEAR MESHING ANALYSIS

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Summary

The project involved developing a mathematical model of machining teeth of a spiral bevel gear and a mathematical model of the gear pair. The mathematical model of machining was based on generative machining with a single indexing system (face milling). On the basis of tool geometry, technological settings, kinematics of the process based on the vector and matrix calculus and differential machining geometry, a tooth model was built and tooth surfaces were obtained. The mathematical model of the gear pair was developed with the use of the gear geometry and the obtained tooth surfaces of the pinion and the gear. The model allows for the possibility of introducing errors due to gear settings and tolerances of the manufacturing errors in housings and other transmission components. The mathematical model of the gear pair was used to obtain the contact pattern and the transmission error graph. An analysis of the results and the application of meshing quality indicators allowed us to improve the gear transmission. This process was carried out in an iterative cycle by changing the set-up (by modifying technological machining) parameters of the machined surfaces of the teeth. The application of both models, i.e. the mathematical model of machining teeth of a spiral bevel gear and a mathematical model of the gear pair, was presented using the example of aircraft gear 18:43.

Keywords: spiral bevel gear, tooth contact analysis (TCA).

MATEMATYCZNY MODEL OTRZYMYWANIA POWIERZCHNI ZĘBÓW KÓŁ STOŻKOWYCH O KOŁOWO-ŁUKOWEJ LINII ZĘBA ORAZ ANALIZY WSPÓŁPRACY PRZEKŁADNI

Streszczenie

W ramach zadania opracowano matematyczny model nacinania uzębienia kół stożkowych o kołowo-łukowej linii zęba oraz model matematyczny przekładni konstrukcyjnej. Model matematyczny nacinania uzębienia dotyczy obróbki obwiedniowej z podziałem przerywanym (face milling). Na podstawie geometrii narzędzia, ustawień technologicznych obrabiarki oraz kinematyki obróbki w oparciu o rachunek wektorowo-macierzowy oraz geometrię różniczkową zbudowano model nacinania uzębienia i otrzymano powierzchnie zębów kół. W oparciu o geometrię przekładni oraz uzyskane powierzchnie zębów kół zbudowano model przekładni konstrukcyjnej z możliwością wprowadzenia błędów wynikających z ustawień kół oraz tolerancji wykonawczych korpusów i pozostałych elementów przekładni. Model przekładni konstrukcyjnej służy do otrzymywania śladu współpracy oraz określania nierównomierności przekazywania ruchu. Na podstawie wymienionych wskaźników jakości zazębienia, następuje dopracowanie przekładni, które odbywa się w cyklu iteracyjnym przez zmianę parametrów ustawczych nacinania powierzchni zębów kół. Zastosowanie obu modeli przedstawiono na przykładzie przekładni lotniczej 18:43.

Słowa kluczowe: stożkowe koła zębate o kołowej linii zęba, analiza zazębienia przekładni.

1. INTRODUCTION

Contemporary bevel gear design process is based on virtual simulation of gear machining, followed by verifying their meshing in a design pair. Since contact pattern is the main indicator of correct gear mating, its location on tooth flank, shape and size are of critical importance. Therefore, the simulation of the designed meshing or the simulation of the test gear must allow for manufacturing errors and assembly errors in the gear assembly. If the desired contact pattern is absent, suitable modifications of technological parameters are introduced in the model generating tooth flank surfaces. Such modifications of gear tooth flank geometry allows us to find a satisfactory contact pattern [1][2][7][10].

Through the application of mathematical models of gear tooth cutting and tooth meshing one may eliminate many activities traditionally performed by multiple cuttings of test gears and checking their meshing on test machines. This leads to a significant reduction of time and costs allocated to the introduction of a new bevel gear pair into production. At the same time, the correct meshing can be obtained as early as in the phase of the first physically cut gear pair. This, however, requires high competence in one of commercially available computer-aided design systems (and purchase thereof) or developing a suitable software tool. One of the possibilities in this area is to design a numerical machining model. Correctly reproduced (in a model) engineering systems and cutting kinematics provide the basis for verifying the meshing of a design pair and/or making modifications to gear teeth in order to arrive at a suitable contact area and smooth performance.

To this end, a mathematical engineering model was developed, simulating the kinematics of machining on a conventional machine tool and a CNC machine tool Phoenix 175HC by Gleason, taking into account all machining methods.

2. A MATHEMATICAL MODEL FOR DESIGNING TOOTH FLANK SURFACE IN A BEVEL GEAR

A mathematical model of a machine tool was developed as set of dextrorotatory Cartesian coordinate systems replicating subassemblies in a real machine tool. The systems were marked by the letter "S” followed by a subscript denoting the subassembly represented by the system (Fig. 1). The applied set of coordinate systems represents a conventional machine tool for cutting bevel teeth with a circular pitch line (e.g. 116G milling machine, 463 grinding machine). In order to develop a universal machining model, we must consider tooth flank geometry obtained by means of the envelope method, including shape-forming, in which the tooth flank is fully tangential to the direction of tool action. In envelope machining, tooth flank constitutes the envelope of the surface of action, obtainable from an equation system (1.1) composed of an equation family of the surface of action (1.2) and a meshing equation defined on the basis of the kinematic envelope theory by F. Litvin [4].

$$\begin{cases} \boldsymbol{r_1}\left(s_t,\theta_t,\psi_t\right) \\ \boldsymbol{n_1}\cdot\boldsymbol{v_1}^{1t}\left(s_t,\theta_t,\psi_t\right)=0 \end{cases} \tag{1.1}$$

$$\boldsymbol{r_1}\left(s_t,\theta_t,\psi_t\right)=\boldsymbol{M_{1t}}\left(\psi_t\right)\cdot\boldsymbol{r_t}\left(s_t,\theta_t\right) \tag{1.2}$$

where: ψ_t - motion parameter, $\boldsymbol{n_1}$ - unit vector normal to the tool surface determined in the S_1 system, $\boldsymbol{v_1}^{1t}\left(s_t,\theta_t,\psi_t\right)$ - speed of tool relative to the gear in the S_1 system, $\boldsymbol{r_t}\left(s_t,\theta_t\right)$ - vector equation of the surface of action in the S_1 system, $\boldsymbol{M_{1t}}\left(\psi_t\right)$ - transformation matrix in the form of a product of matrices reflecting individual rotations and shifts between input systems in the technological bevel gear model.

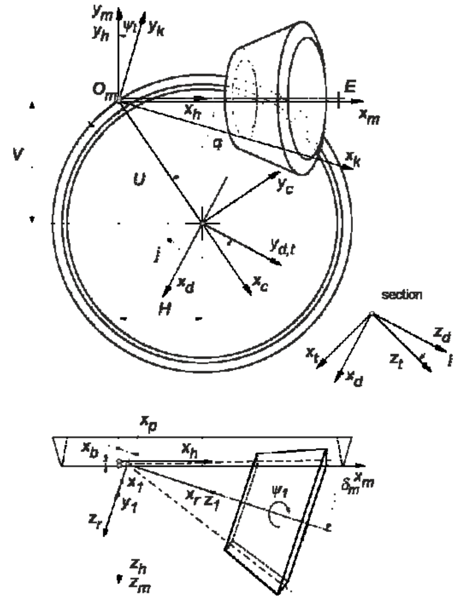

Fig. 1. The location of the machined gear in the technological model together with a set of coordinate systems for machine tool subassemblies reflecting the set-up of technological parameters

A parametric record of the surface of action in a coordinate system related to tool S_t is presented by means of equation (1.3), where as the unit vector normal to these surfaces was provided in formula (1.4):

$$\boldsymbol{r_t}(s_t,\theta_t)=\begin{bmatrix}\cos\theta_t\cdot(r_i\pm s_t\sin\alpha_i)\\ \sin\theta_t\cdot(r_i\pm s_t\sin\alpha_i)\\ -s_t\cos\alpha_i)\end{bmatrix} \tag{1.3}$$

$$\boldsymbol{n_t}(\theta_t)=\begin{bmatrix}-\cos\theta_t\cdot\cos\alpha_i\\ -\sin\theta_t\cdot\cos\alpha_i\\ \mp\sin\alpha_i\end{bmatrix} \tag{1.4}$$

where: s_t,θ_t - parameters related, respectively, to the length of the cutting edge and the formation of the rotary conical surface (Fig. 2); α_i, r_i - cutters' profile angle and the radius of the cutter head (i=wk, wp), the "wk” subscript refers to the concave side of the tooth flank machined by the tool's outer cutters, while the "wp” subscript refers to the convex side of the tooth flank machined by the tool's inner cutters ("upper" symbol for outer cutters, "lower” for inner cutters).

An elaborate set of coordinate systems modelling the technological gear, to which nine basic setting parameters of the machine tool are input (excluding the geometry of the tool and the machined gear), does not allow us to present the meshing equation in an explicit way, i.e. in the form of two-variable parametric equation [9]. In this situation, the resulting tooth flank surface is a set of points, with suitable indices, constituting the solution to the system of equations (1.1). The coordinates of the sought points correspond to the coordinates of the points distributed across the reference grid located in the axial section of the machined gear within the limits of the tooth's active height. The surface is stretched across the resulting set of points by interpolation. Thus, the interpolated tooth flank is subjected to further analysis in the design gear model [9].

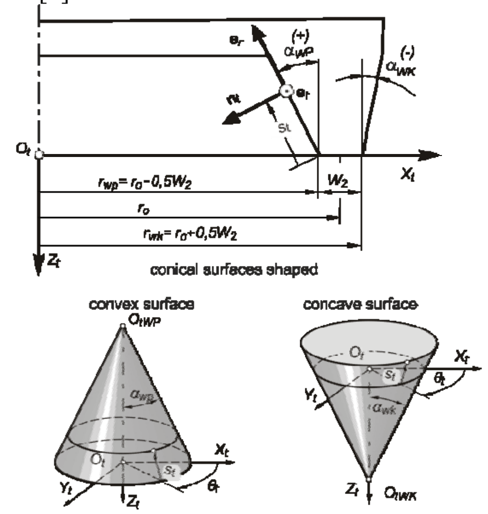

Fig. 2. Geometry of conical surfaces forming concave and convex tooth flanks [6]

3. A MATHEMATICAL MODEL OF THE DESIGN BEVEL GEAR

A mathematical model of the design gear was created for the purpose of tooth contact analysis (TCA) of the gear in unloaded condition. The application of the vector and matrix calculus enables a neat and generalized record of defined geometrical elements, and presents the interdependencies between such elements in a transparent manner. Gear geometry is defined by a set of orthogonal, dextrorotatory coordinate systems reflecting the position of the pinion relative to the gear and allowing for input of assembly errors (Fig. 3). In Fig. 3, systems S_1 and S_2 are related respectively to the pinion and the gear, while other systems determine the mutual position of the mating gears. The engagement is assessed in reference to a fixed coordinate system S_f related to the transmission housing.

In a theoretical design gear model (without the inclusion of assembly errors), meshing quality is assessed by generating contact patterns and the transmission error graph. If the results of gear contact assessment are unsatisfactory, new tooth flank geometry resulting from technological parameter modifications is introduced, and a suitable solution that could provide satisfactory meshing quality is sought. The design gear model also includes the following changes in gear locations: hypoid offset of the pinion (V), axial shift of the pinion (H), axial shift of the gear (J), and the change in the axial intersection angle $\Delta\Sigma$. This extended model of the design gear allows us to evaluate the effect of assembly errors on qualitative gear performance indicators [8].

The use of the geometry of pinion and gear flanks as the input in the mathematical model of the design gear makes it possible to obtain additional information about tooth contact (TCA) in an unloaded gear. These include path of contact and the ease-off graph [2][4].

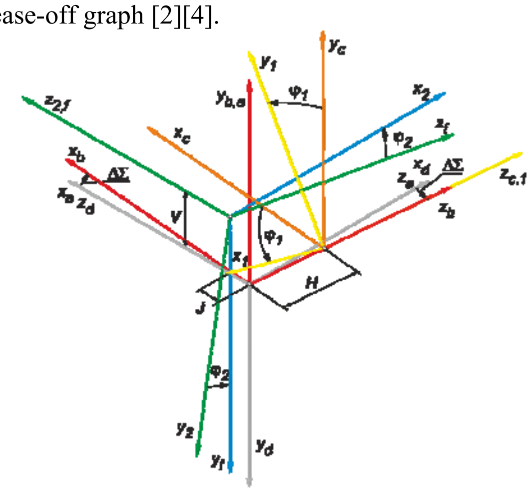

Fig. 3. Design gear model with deviations as applied: in the analysis of the effect of tooth geometry on gear meshing quality (for zero deviations); and to determine the gear's sensitivity to assembly and manufacturing errors

4. THE RESULTS OF THE APPLICATION OF THE MATHEMATICAL MODEL OF GEAR TOOTH DESIGN AND THE MATHEMATICAL MODEL OF A DESIGN BEVEL GEAR

The analysis was performed for a 18:43 gear, whose basic geometry and setting parameters of machining and tool geometry are presented in Tables 1, 2 and 3.

Table 1. Basic geometrical data of the 18/43 gear

Quantity	**Desig-nation**	**Pinion**	**Gear**
Number of teeth	z	18	43
Hand of spiral		Right	Left
External transverse module	m_t		2.163[mm]
Pressure angle	α	20°	
Shaft angle	Σ	70°	
Spiral angle	β	35°	
Mean cone distance	R	51.688 mm]	
Face width	b	16.260[mm]	
External whole depth	h	4.502[mm]	4.502[mm]
Clearance	c	0.40[mm]	0.40[mm]
External height of addendum	h_a	2.110[mm]	1.990[mm]
External height of tooth root	h_f	2.392[mm]	2.512[mm]
Pitch angle	δ	18°59'17"	51°0'42"
Dedendum angle	θ_f	2°58'12"	3°48'0"
Addendum angle	θ_a	3°48'0"	2°58'12"

Table 2. Geometrical tool data

Tool parameters		pinion (concave)	pinion (convex)	gear (both flanks)
D_{0R}	Diameter of cutter head	152.566 [mm]	154.990 [mm]	152.4 [mm]
W_2	Width of the blade tip	0.664 [mm]	0.664 [mm]	1.145 [mm]
r_0	Fillet radius	0.381 [mm]	0.381 [mm]	0.459 [mm]
α_{wk}	Cutter pressure angle (outer)	16 [°]	24 [°]	20 [°]
α_{wp}	Cutter pressure angle (inner)	20 [°]	13 [°]	20 [°]

Table 3. Basic setting data for the 18:43 gear pair

Basic machine settings	pinion (concave)	pinion (convex)	gear (both flanks)
Cradle angle	83°55'25"	81°30'23"	82°48'57"
Radial distance	62.723 [mm]	64.356 [mm]	62.913 [mm]
Hypoid offset	-0.05 [mm]	0.04 [mm]	0.00 [mm]
Machine root angle	16°01'05"	16°01'05"	47°12'42"
Machine centre to back	-0.110 [mm]	0.576 [mm]	0.00 [mm]
Sliding base	0.734 [mm]	0.552 [mm]	1.459 [mm]
Tilt angle	0°	0°	0 [°]
Swivel angle	0°	0°	0 [°]
Roll ratio	0.326333	0.322366	0.778996

Based on the above geometrical and set-up data, the mathematical tooth cutting model was applied to find the solution to the equation system (1.1) defining the envelope of the tool's surface of action. The solution to the equation system is numerical, therefore gear and pinion flanks are presented as sets of points. To facilitate calculations and further analyses a reference grid was introduced, evenly dividing the tooth flank within the active height range [5]. In addition, the reference grid verifies if tooth undercut occurs. Tooth surfaces of the gear and the pinion generated in the Mathcad software were presented in the form of a grid of points and the interpolated surface in Fig. 4 (gear) and 5 (pinion).

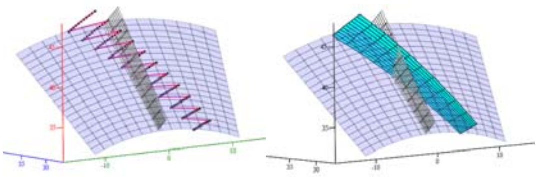

Fig. 4. Grid of points (left-hand drawing) representing the numerical solution to the envelope equation (1.1) – defining the concave surface of gear tooth flank (the right-hand drawing shows the interpolated surface)

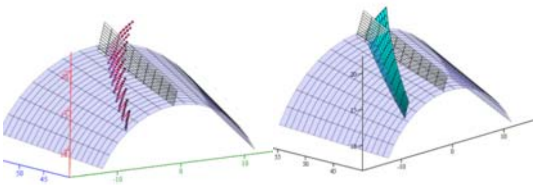

Fig. 5. Grid of points (left-hand drawing) representing the numerical solution to the envelope equation (1.1) – defining the convex surface of pinion tooth flank (the right-hand drawing shows the interpolated surface)

Pinion and gear surfaces obtained from the mathematical cutting model were compiled in the design gear model. A 6 mm-thick virtual marking compound was assumed for determining the contact pattern. For the presented data, contact path, momentary and summary contact patterns and transmission error graph were generated (Fig. 6, 7, 8, 9).

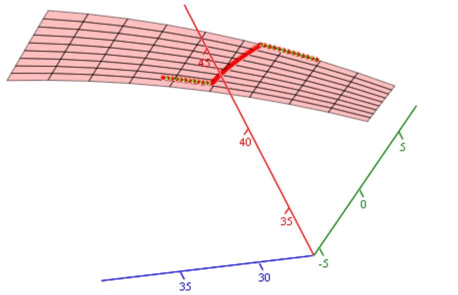

Fig. 6. Contact path illustrated on the concave surface of the gear flank

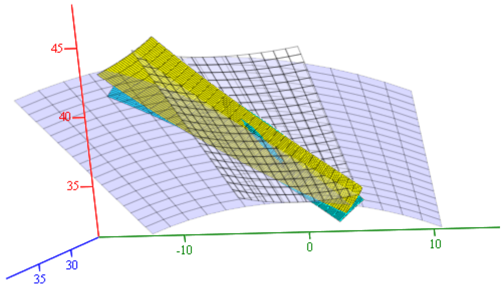

Fig. 7. Momentary gear contact pattern (yellow – pinion, turquoise – gear, other parts of pinion and gear reference cones)

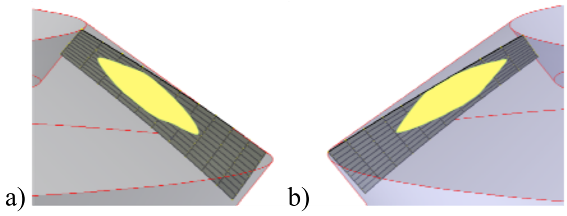

Fig. 8. Reference summary contact pattern for a 18:43 transmission on the drive side (a) and coast side (b) of the left-hand gear generated on the basis of data presented in Tables 1, 2 and 3

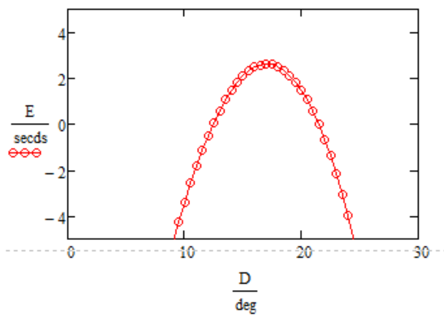

Fig. 9. Transmission error graph

The analysed unloaded gear achieves the correct contact pattern both on its drive and coast side, i.e. the pattern is focused, slightly shifted in the direction of small modules and does not reveal any edging. As expected, the transmission error graph is also of parabolic shape, providing uniform and smooth gear performance, and the maximum deviation at motion transmission point is 5 seconds of arc. Based on the above gear meshing quality criteria, the present gear design is considered correct.

Table 4. Offset values for transmission configuration parameters

		Assembly errors			
		Hypoid offset V [mm]	Axial shift of pinion H [mm]	Axial shift of gear J [mm]	Shaft angle change ΔΣ [min]
Instance	a)	-0.10	0.00	0.00	0
	b)	0.10	0.00	0.00	0
	c)	0.00	-0.10	0.00	0
	d)	0.00	0.10	0.00	0
	e)	0.00	0.00	-0.10	0
	f)	0.00	0.00	0.10	0
	g)	0.00	0.00	0.00	-5
	h)	0.00	0.00	0.00	5

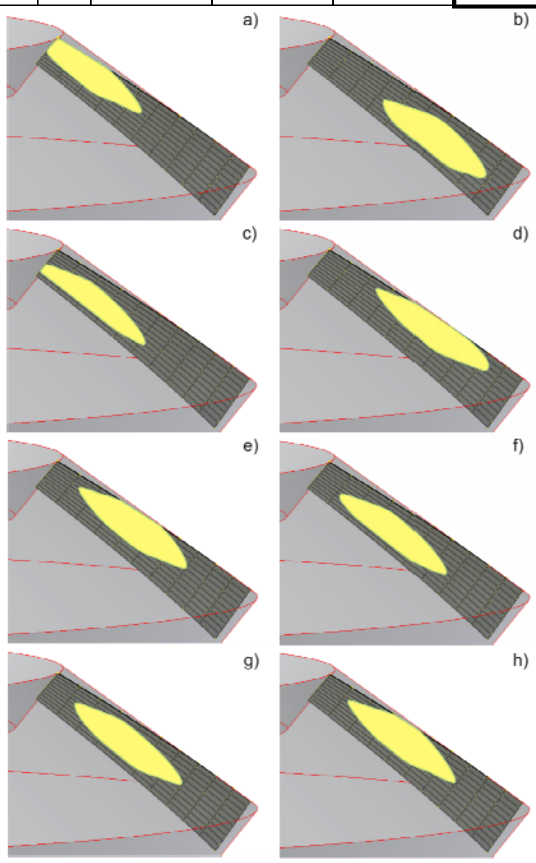

Fig. 10. Summary contact patterns for a 18:43 transmission on the drive side of the gear tooth (left-hand gears) for the assumed deviations (positive and negative) V, H, J and ΔΣ

Having obtained satisfactory gear meshing indicators, i.e. a suitably shaped, sized and positioned contact pattern, and with a transmission error of up to 10 seconds of arc, the gear should be assessed in terms of its sensitivity to assembly errors within tolerance limits [4]. To this end, tooth surfaces of the wheel and the pinion for the drive side and the coast side were compiled, taking into account axial shifts defined by parameters: V, H, J and ΔΣ. Offset values for the parameters for which the position of the contact pattern is to be verified are usually defined by the designer. Their levels result from the tolerances assumed in the performance of the assembly components as well as the gear's load conditions. In order to perform the analysis of the gear's sensitivity to assembly errors, V, H, J and ΔΣ values listed in Table 4 were assumed.

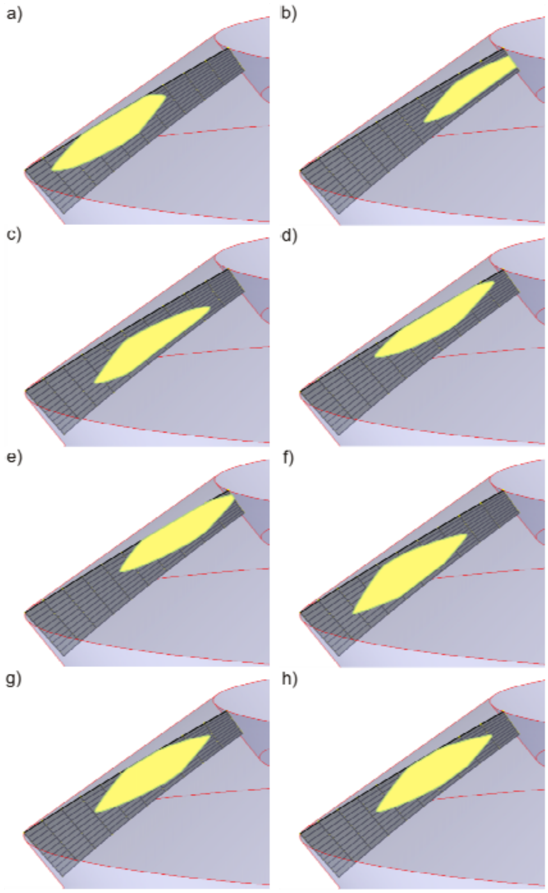

Fig. 11. Summary contact patterns for a 18:43 transmission on the coast side of gear tooth (left-hand gears) for the assumed deviations V, H, J and ΔΣ.

Considering the results of the analysis of the gear's sensitivity to assembly errors, it must be concluded that the deviation of the axial intersection angle is very small and has only a slight effect on the change of the position of the contact pattern for this gear. It will affect the amount of backlash and tip clearance. Negative hypoid offset V and axial shift of the pinion H for the drive side of the engagement lead to an unacceptable contact pattern, mainly due to edging. A similar situation occurs for the coast side – in this case for positive values of hypoid offset and axial shift of the pinion, the contact pattern is too narrow and leads to edging. A gear wheel assembly error in the form J axial shift is less sensitive than vertical and horizontal axial shifts of the pinion. The contact pattern shifts primarily in the direction of tooth height and only slightly in the direction of tooth width.

5. CONCLUSIONS

Mathematical models of tooth cutting and design gear engagement play an important role in designing spiral bevel gears. They allow us to determine the correct selection of tool geometry and technological machining parameters. Next, by compiling the resulting pinion and gear wheel tooth surfaces in a design gear model and their suitable penetration, mathematical models make it possible to generate contact pattern – the main indicator of meshing quality – and transmission error graph. If meshing performance is unsatisfactory, by changing the geometry of the pinion and the gear wheel resulting from technological settings of the machining process and tool geometry, we may effectively shape the form of the contact pattern and influence the accuracy of transmission. With the correct meshing performance in place, an analysis of the sensitivity of the gear to performance and assembly errors must be carried out, taking into account operating conditions of the gear. In this way we may predict gear performance in loaded condition. The awareness of these meshing indicators in unloaded condition enables the reduction of the design period and prototype-making time, which means lower manufacturing costs.

After making improvements to the geometry of gear flanks using TCA (i.e. having arrived at a contact pattern of expected shape and position), a loaded gear tooth contact analysis (LTCA) must be performed. LCTA may be carried out using the finite element method (FEM) [1][3]. Models simulating gear load are also created using tooth flanks obtained from the mathematical model of tooth machining. If results are satisfactory, the production of prototypes should follow. For aeronautics applications, tests in real conditions (RIG test) must be carried out.

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