FORCE IDENTIFICATION WITH USE OF SPATIAL FILTER BASED ON ODS

Krzysztof MENDROK

AGH University of Science and Technology, Department of Robotics and Mechatronics,
al. Mickiewicza 30, 30-059 Krakow, Poland, e-mail: mendrok@agh.edu.pl

Summary

In last few decades it can be observed that there is a significant growth of the interest in the structural health monitoring (SHM) systems development and applications. Unfortunately many authors focuses only on the damage detection and other activities related with diagnosis of fault. Meanwhile, classical SHM system by definition should have in addition to a diagnostic module also module for load monitoring. This load can be measured, but easier and cheaper is to identify it from the measured response of the object. Often it is the only practical possibility to monitor the excitation. The paper presents a trial to apply a spatial filter based on operational deflection shapes (ODS) to force identification. The idea of spatial filter will be shown together with the method of force reconstruction. The simulation verification and comparison with classical modal filter will be also provided.

Keywords: Force identification, spatial filter, modal filter, operational deflection shapes

IDENTYFIKACJA SIŁY Z ZASTOSOWANIEM FILTRU PRZESTRZENNEGO OPARTEGO NA ODS

Streszczenie

W kilku ostatnich dziesięcioleciach można zaobserwować znaczny wzrost zainteresowania budową i zastosowaniami układów monitorowania stanu obiektów (ang. Structural Health Monitoring - SHM). Niestety większość autorów skupia się na zagadnieniach wykrywania uszkodzeń i innymi czynnościami związanymi z diagnostyką. Tymczasem, klasyczny układ monitoringu powinien, z definicji, posiadać poza modułem diagnostycznym również moduł odpowiedzialny za monitorowanie obciążeń. Te obciążenia mogą być mierzone, lecz taniej i łatwiej jest identyfikować je na podstawie pomiaru odpowiedzi obiektu. Często jest to jedyna praktyczna możliwość ich monitorowania. Artykuł przedstawia próbę zastosowania filtru przestrzennego opartego na eksploatacyjnych formach drgań (ang. Operational Deflection Shapes - ODS) do identyfikacji wymuszeń. Pokazana będzie idea filtru przestrzennego wraz z metodą jego aplikacji do rekonstrukcji siły. Zawarta będzie także weryfikacja symulacyjna i porównanie z klasycznym filtrem modalnym.

Słowa kluczowe: Identyfikacja sił, filtr przestrzenny, filtr modalny, eksploatacyjne formy drgań

1. INTRODUCTION

Structural health monitoring (SHM) is a relatively new appearance in science. The first references to this subject appeared in world literature in the 1980s. SHM is a natural development of non-destructive testing (NDT) and condition monitoring (CM). According to its definition the SHM is: the interdisciplinary field of science leading to the provision of, at any moment of the working life of the object, a diagnosis of the material integrity of successive elements, as well as the state of all elements together creating the tested object as a whole. This state must stay in the range defined during design of the object, although it may change as a result of normal usage, environmental effects or unexpected events. Thanks to the continuous monitoring, which allows an analysis of the complete history of the structural health, as well as the monitoring of operating conditions (loads), the SHM system should also provide a prognosis (damage development, remaining work time etc.) [1]. As it was mentioned in the abstract, the second part of the definition, which says about the excitation monitoring is often forgotten. And in author opinion it is equally important as damage detection in the SHM systems. In Figure 1 the SHM system block diagram is presented.

The presented block diagram depictures that each SHM system should be composed of three equally important modules:

– a diagnostics module,
– a module monitoring operating conditions,
– a database containing material models and damage accumulation models.

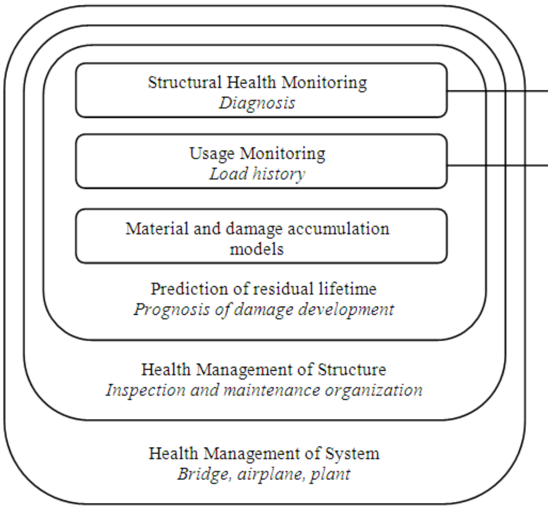

Fig. 1. Block diagram of SHM system

Last two of the above modules requires the load monitoring. Unfortunately measurement of operational excitations is sometimes very difficult or even impossible. That is why the excitations are often monitored on the basis of structure response measurement. The actual excitation value is reconstructed with use of the inverse problem solution.
One of the method for force identification is the application of modal filter.

2. FORCE IDENTIFICATION WITH USE OF MODAL FILTER

The modal filter is a tool for extracting the modal coordinates of each individual mode from the system outputs by mapping the response vector from the physical space to the modal space [2]. It is done by means of a new modal parameter: reciprocal modal vectors. Reciprocal modal vectors should be orthogonal with respect to the modal vectors ψ_r, and thanks to this condition, they can be applied to the decomposition of the system responses to modal coordinates η_r

$$\eta_r(\omega) = \psi_r^{\,T} \cdot \{x(\omega)\} = \left(\frac{\{\phi_r\}^T}{j\omega - \lambda_r} + \{\psi_r\}^T \{\phi_r^*\} \frac{\{\phi_r^*\}^T}{j\omega - \lambda_r^*} \right) \{f(\omega)\} \tag{1}$$

where: ϕ_r – the r-th modal vector, λ_r – the r-th pole of the system, $\{x\}$ – vector of system responses (output), $\{f\}$ – vector of excitation forces (input)

Application of the modal filter to force identification proceeds in four major steps [3]:
1. Transfer the outputs of the system from physical coordinates to modal coordinates using modal filters.
2. Determine the number of uncorrelated system inputs based on the weighted modal coordinates.
3. Locate these unknown inputs.
4. Calculate the amplitude of these inputs.

Denoting $\eta_r(\omega)(j\omega - \lambda_r)(j\omega - \lambda_r^*)$ by $\hat{\eta}_r(\omega)$ excitation forces can be calculated from the following formula:

$$\frac{\hat{\eta}_r(\omega)}{\lambda_r^* - \lambda_r} = \{\phi_r\}^T \{f(\omega)\} \tag{2}$$

Or, in matrix form:

$$[F] = [\Phi]^{T+} [\Pi] \tag{3}$$

where:

$$[\Pi] = \begin{bmatrix} \frac{\hat{\eta}_1(\omega_A)}{\lambda_1^* - \lambda_1} & \cdots & \frac{\hat{\eta}_1(\omega_Z)}{\lambda_1^* - \lambda_1} \\ \vdots & \ddots & \vdots \\ \frac{\hat{\eta}_n(\omega_A)}{\lambda_n^* - \lambda_n} & \cdots & \frac{\hat{\eta}_n(\omega_Z)}{\lambda_n^* - \lambda_n} \end{bmatrix} \tag{4}$$

$$[F] = [f(\omega_A) \cdots f(\omega_Z)]$$

Note that the dimension of the input vector *{f(ω)}* is defined to be the same as that of the modal vector. This is why many of its rows must be zero, except those corresponding to the vibration sources. That is, the rank of the matrix *[F]* equals the number of uncorrelated input forces. Since *[Φ]* is a full rank matrix, the rank of *[Π]*, which contains the weighted modal coordinates should be the same as the rank of *[F]*. As a result the number of vibration sources can be determined by inspecting the singular values of the matrix *[Π]*.
The method is quite accurate and gives good representation of identified force of random or sine wave type [4] but it has the serious disadvantage. This drawback is the need for knowledge of the modal model to determine the modal filter coefficients. Such a model can be obtained by using the active modal test, and this is often troublesome for objects that cannot be isolated from environmental or operational excitation, or are too large and rigid that they can be effectively enforced (bridges, towers, buildings). It is possible to eliminate this disadvantage by the use of a spatial filter based on ODS, instead of modal vectors.

3. ODS BASED SPATIAL FILTER

At first, it should be reminded what an ODS is. In [5] one can find the following definition: "ODS has been defined as the deflection of a structure at a particular frequency". Different types of data can be acquired from a measurement, both in time and frequency domain for ODSs determination. The authors focused on frequency domain data and the possible selection included [5]: linear spectra, auto power spectra, cross power spectra, FRFs and ODS FRFs.
The formulation of the spatial filter was described in [6]. In a classical approach, construction of the modal filter requires finding a single vector *{ψ$_r$}*

that will be orthogonal to all modal vectors $\{\phi_k\}$ but r-th – $\{\phi_r\}$, thus the filtration (performed by calculating dot product between the filtering vector and data) will cancel out the contribution of all modes except the r-th, providing a function of a single mode only. Important difference between modes and ODSs is that modes are strictly related to natural frequency of the structure whereas ODSs are defined for any frequency, stating a problem of selection of particular vectors that are suitable for filter construction. In the presented method, similarly to the classic technique mentioned above, only these ODSs were taken into account, which correspond to the natural frequencies and possibly rotational velocity harmonics of an object and they will be selected by the method of peak picking. In real life application, excitation forces will strongly influence system responses, therefore presenting a challenge in proper ODSs selection, but in this particular case this matter seems to pretty straightforward.

Mathematical criteria needed to be met in order to find a proper filtering vector $\{\psi_{ODSr}\}$ tuned to extract only contribution of mode corresponding to r-th ODS is presented by Eq. 1 [2]. However, in this case $\{\phi_{ODSk}\}$ denotes k-th ODS vector. Assuming that none of the following vector is zero length, $\{\{\psi_{ODSr}\}$ must be orthogonal to all ODS vectors but $\{\phi_{ODSk}\}$

$$\{\phi_{ODS\,k}\}^T\{\psi_{ODS\,r}\} = \begin{cases} 1, & r = k \\ 0, & r \neq k \end{cases} \tag{5}$$

Equation (5) can be expanded into a set of equations (6), and then be solved with respect to $\{\psi_r\}$ in order to find a proper filtering vector.

$$\begin{bmatrix} \{\phi_{ODS\,1}\} \\ \vdots \\ \{\phi_{ODS\,n}\} \end{bmatrix} \{\Psi_{ODS\,r}\} = \{\rho_r\}, \quad \text{where}$$

$$\{\rho_r\}^T = \{a_1 \dots a_k\} \quad a_i = \begin{cases} 1, & r = k \\ 0, & r \neq k \end{cases} \tag{6}$$

At this point it should be stated that the number of equations and unknowns determine solvability of the system. If there are less equations than unknowns, that is, less responses measured than ODSs taken, it is impossible to find a vector that would be orthogonal to all but one ODS. Such a problem can be overcome by limiting band of a spectra taken into consideration to a one, which has just as many peaks as response points. This approach seems to be relevant especially in real life application where physical structures have infinite number of DOFs and there is a limited number of sensors. In the case presented in this paper ODS matrix is square and invertible (providing that ODS vectors are linearly independent), therefore only one exact solution exists and can be found by solving equation (6).

The spatial filtration is done by multiplication of vector $\{\psi_{ODSr}\}^T$ by the response spectra matrix, as stated in the equation (7).

$$\eta(\omega) = \{\Psi_{ODS\,r}\}^T \cdot \begin{bmatrix} \{x_1(\omega)\}^T \\ \vdots \\ \{x_n(\omega)\}^T \end{bmatrix} \tag{7}$$

where $\eta(\omega)$ denotes ODS filter output and $x_i(\omega)$ is power spectral density of i-th response point.

4. SIMULATION VERIFICAITON OF THE SPATIAL FILTER OPERATION

Simulation verification of the described spatial filter was shown in [6]. Here the process is repeated but on the model used for force identification. For this purpose a model of linear, time-invariant, mass-damper-spring mechanical system was defined, its scheme is showed in Figure 1.

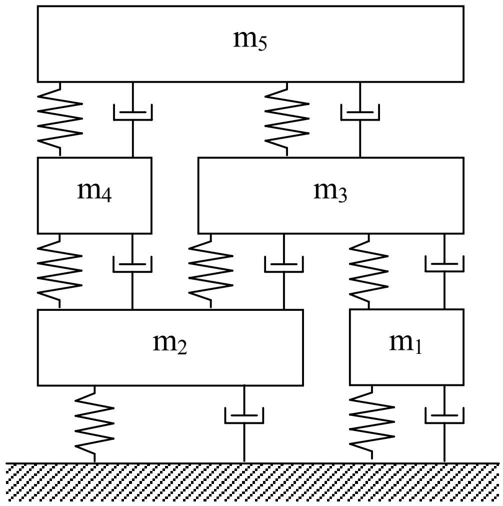

Fig. 1. Model used for simulation

The model consists of five masses interconnected with each other using spring (proportional stiffness) and damper (proportional damping) elements. The physical parameters of the system are gathered in Table 1.

Table 1. Physical parameters of the model

Mass [kg]	$m_1 = 4$; $m_2 = 1$; $m_3 = 3$; $m_4 = 2$; $m_5 = 5$;
Damp. coeff. [N s / m]	$c_{01} = 3.75$; $c_{02} = 2.5$; $c_{13} = 1.5$; $c_{23} = 4.2$; $c_{24} = 1.3$; $c_{35} = 1.5$; $c_{45} = 3.7$;
Stiff. coeff. [N / m]	$k_{01} = 20000$; $k_{02} = 40000$; $k_{13} = 25000$; $k_{23} = 30000$; $k_{24} = 50000$; $k_{35} = 40000$; $k_{45} = 25000$;

The output of the system are the displacements of all masses. Band noise type excitation force was applied to mass number 3. Selection of wide-band input signal reassures that the object under scope is well excited and contribution of all modes will be seen in the output of the system. In the simulation zero initial conditions were specified, Hanning window was applied to the output data. The eigenvalue problem for this system was solved, modal parameters are shown in Table 2.

Table 2. Dynamic properties of examined model

Mode no.	Natural frequency [Hz]	Damping coefficient [%]
1	8.3	0.27
2	16.5	0.53
3	26.6	0.91
4	30.2	0.58
5	58.6	1.15

In this particular case ODS created from Power Spectra Densities (PSD) responses was chosen for the construction of a spatial filter. Welch estimator was utilized to obtain PSD of each output from the time displacement vector of each mass. Results of the estimation of PSD of the responses are presented in Figures 2 in form of waterfall plot.

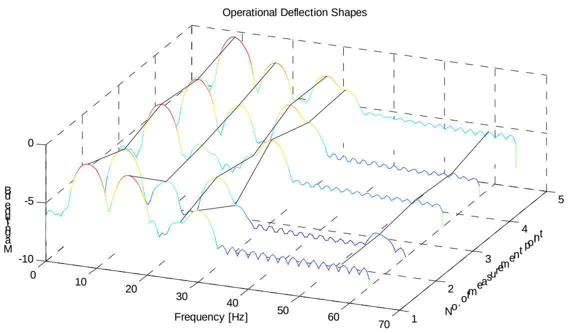

Fig. 2. PSDs of the system output

Thick black lines mark the ODS vectors at natural frequencies of the structure. These vectors will create an ODS matrix that will be used for the construction of the spatial filter. Number of the degrees of freedom of the system is equal to the number of its modes or natural frequencies, therefore the count of ODS vectors chosen will match the size of each vector, forming a square ODS matrix.

For the model presented in the beginning of the chapter, five filters, tuned to ODS at each natural frequency of the system, where constructed and applied. The first three outputs of the filters are illustrated in Figure 3.

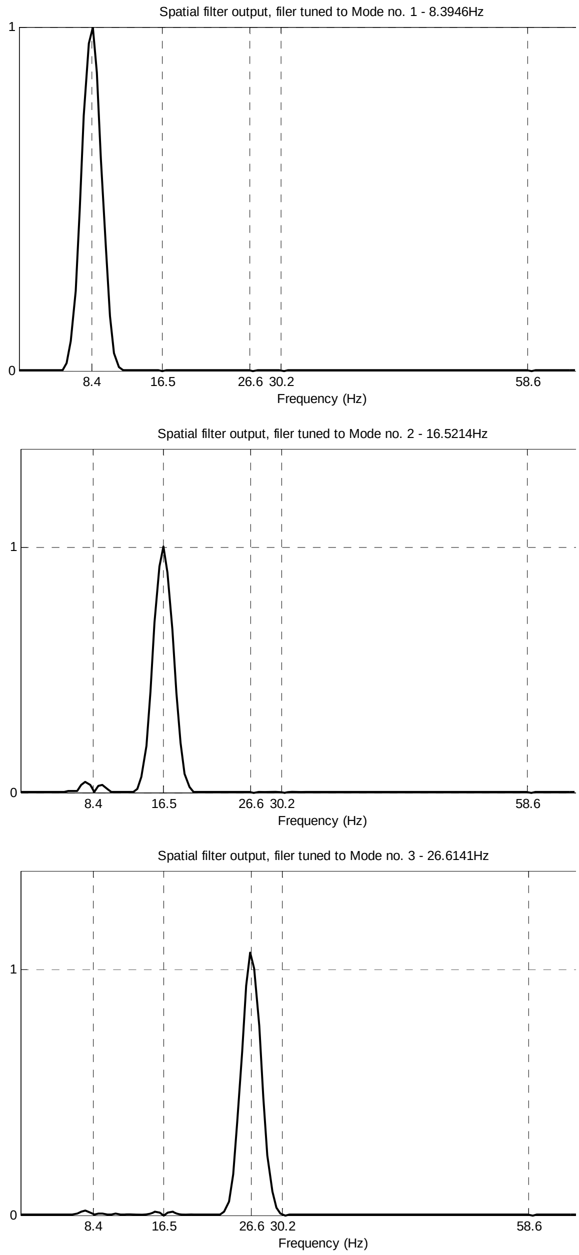

Fig. 3. First three outputs of spatial filter tuned for ODS corresponding to natural frequency of the system

In the output of the filters number 1, 2, 3, 4 a single peak at the frequency to which the filter was tuned is visible. Some minor residual traces of can be spotted in the remaining frequency bands, however their amplitude does not affect the general image. Isolated single ODS can be easily traced revealing changes of in the structure, such as damage occurrence. Output of remaining filter 5 is quite distorted and can be a reason for disturbance in force identification.

Comparison of filtration results obtained for classical modal filter and this new approach was presented in [6], and proved its good efficiency. The method was also successfully applied for damage detection [7].

5. APPLICATION OF SPATIAL FILTER FOR FORCE IDENTIFICATION

In the consecutive step the simulation model described in the previous chapter was used for force identification. For that purpose the force signal was applied in the mass no. 3. The responses in form of vibration displacements were calculated for all of the masses. The excitation force was a sum of random noise with zero mean and amplitude 1 and sine wave of frequency 21 Hz and amplitude 2. Next the force identification algorithm presented in Chapter 2 was changed to work for spatial filter based on ODS. The modification was simple and can be described by the following formulas:

$$\eta_r(\omega) = \psi_{ODS\,r}^{\;T} \cdot \{x(\omega)\} \tag{8}$$

$$\frac{\hat{\eta}_r(\omega)}{\lambda_r^* - \lambda_r} = \{\phi_{ODS\,r}\}^T \{f(\omega)\} \tag{9}$$

Or, in matrix form:

$$[F] = [\Phi_{ODS}]^{T+} [\Pi] \tag{10}$$

Results of force identification obtained with use of this algorithm are presented in Figure 4. In Figure 5 the original spectrum of excitation force signal is shown for comparison.

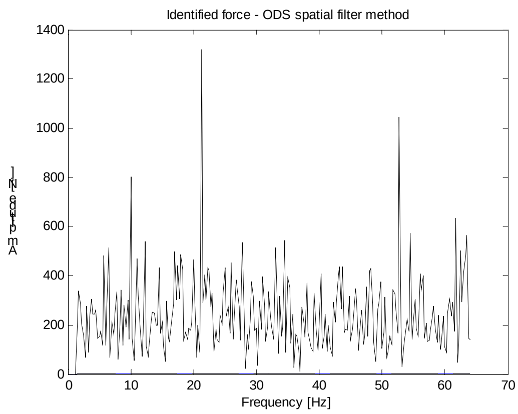

Fig. 4. Excitation force identified with use of spatial filter based on ODS

It was necessary to place both spectra on separate plots, due to the fact that identified force is not correctly scaled. The reason for this scaling problem is the fact that for the spatial filter described in the paper the ODS are used instead of modal vectors. The latter one are scaled and thanks to that independent of excitation. In other words they give a proper amplitude of identified force. The visual comparison reveals that the character of identified spectrum is well represented. To confirm this observation the correlation coefficient was calculated. It value amounts: 0.9893 which proves the effectiveness of applied method.

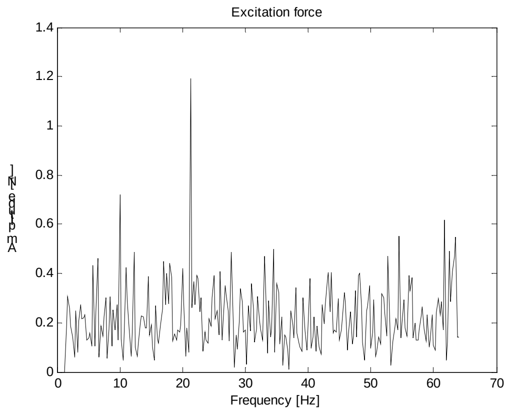

Fig. 5. Original excitation force

In order to scale the identified signal, it is enough to run a simple test with the single sine wave excitation of known amplitude. And in the next step identify this force with use of the method. Having both signals one can calculate the scaling factor as a ratio of peaks amplitude of both spectra. Such a procedure was applied in the described case. The scaling factor amounted 9.01 $*10^{-4}$.

6. COMAPRISON OF FORCE IDENTIFICATION RESULTS OBTAINED WITH ODS FILTER AND MODAL FILTER

The last point of simulation verification presented in the paper is the comparison with results of force identification performed with use of classical modal filter. The summary of the identification results is shown in Figure 6.

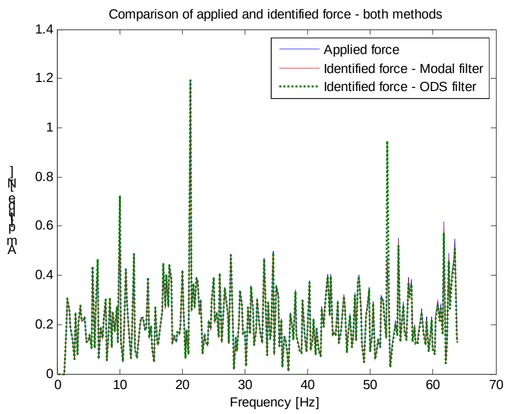

Fig. 6. Comparison of identification results

Figure 6 is not very informative, while both methods (ODS filter method results were scaled) gave very good results. To highlight the differences between the methods, the results of subtraction of the identified from original force is presented in Figure 7.

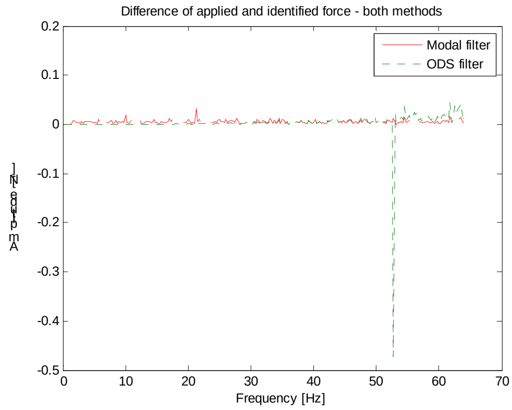

Fig. 7. Difference of applied and identified force for both methods

Figure 7 shows that ODS filter works very well for low frequencies. Up to 30 Hz its performance is even better than the modal filter. Above 50 Hz the situation is reversed. Especially big inaccuracy is visible for the fifth natural frequency. It is caused by the low dynamics of the system response for this frequencies (see Figure 2). Also the ODS filtration results for this filter were the worst. The correlation coefficients calculated between original force and identification results amount: 1 for the modal filter and 0.9893 for the spatial filter based on ODS.

7. SUMMARY

In the paper the force identification algorithm is presented. The method is based on the well known modal filter method. The novelty is in the fact that the modal filter is replaced with the spatial filter based on ODS. It gives the significant advantages with respect to the classical method. It does not require active testing and no modal analysis has to be done. The problems that Author faced during the simulation verification were: scaling and lower accuracy for higher frequencies where dynamics of the responses is significantly lower. Apart from these, method is much easier for application due to the fact that it mainly bases on the operational data. There is only one measurement with known excitation required (in order to scale results) contrary to full modal active test needed for modal filter determination.

ACKNOWLEDGEMENT

Research funding from the Polish research project No. 2011/01/B/ST8/07210 is acknowledged by the authors.

REFERENCES

[1] Balageas D., Fritzen C.-P., Guemes A. (Ed.), *Structural Health Monitoring*, ISTE Ltd., London, UK, 2006,

[2] Zhang, Q., Allemang, R. J., Brown, D. L., *Modal filter: Concept and applications*, Proceedings of 8th IMAC, Orlando, FL, USA, pp. 487–496, 1990,

[3] Shih C. Y., Zhang Q., Allemang R. J., *Force identification by using principle and modal coordinate transformation method*, Vibration Analysis Techniques and Applications, ASME publication DE, vol. 18(4), pp.303-309, 1989,

[4] Mendrok K., Uhl T., *Load identification using a modified modal filter technique*, Journal of Vibration and Control, vol. 16, no. 1, pp. 89–105, 2010,

[5] Schwarz, B. J., & Richardson, M. H., *Introduction to operating deflection shapes*, CSI Reliability Week, Orlando, FL, 1999,

[6] Wojcicki J., Mendrok K., Uhl T., *Spatial filter for operational deflection shape component filtration,* Key Engineering Materials, vols. 569–570, pp. 868–875, 2013,

[7] Wojcicki J., Mendrok K., *Structural changes detection with use of operational spatial filter*, in Proceedings of S*URVEILLANCE 7 International Conference on Acoustical and Vibratory Surveillance Methods and Diagnostic Techniques] : Chartres, France, October 29–30, 2013* ibliogr. s. 65

Krzysztof MENDROK DSc. Eng. is a senior researcher in the Department of Robotics and Mechatronice of the AGH University of Science and Technology. He is interested in development and application of various SHM algorithms. He mainly deals with low frequency vibration based methods for damage detection and inverse dynamic problem for excitation identification.