MULTIPLE DAMAGE LOCALIZATION USING LOCAL MODAL FILTERS

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Summary

The structural health monitoring systems seeks to increase the accuracy of diagnosis. Increasingly, raw information about the occurrence of damage is not sufficient. The aim is to determine its location, identification, and even predictions about its propagation. Based on these requirements already in 1993 Rytter [1] proposed the classification of monitoring systems by dividing them into levels from 1 (detection) to 4 (prognosis). To meet the assumptions a method of fault location using local modal filters was developed [2], which was the extension of damage detection method based on spatial filtering [3]. The method of local modal filters correctly locates damage on the object. However, there is a possibility that the multiple damage can occur in the object. In this paper the author will answer the question whether this method will be effective in this type of situation. In the simulations consecutively damages of varying size and location were introduced in the object and it was checked whether they are detectable, and as their increasing number affects the accuracy of location.

Keywords: modal filtration, damage detection, damage localization.

LOKALIZACJA WIELU USZKODZEŃ Z ZASTOSOWANIEM LOKALNYCH FILTRÓW MODALNYCH

Streszczenie

W układach monitorowania stanu konstrukcji dąży się do wzrostu dokładności stawianej diagnozy. Coraz częściej informacja jedynie o wykryciu szkodzenia jest niewystarczająca. Celem jest lokalizacja, identyfikacja a nawet prognoza jego rozwoju. W oparciu o te wymagania już w 1993 Rytter [1] zaproponował klasyfikacje układów monitorowania dzieląc je na poziomy od 1 (wykrycie) do 4 (prognoza). Aby sprostać tym wymaganiom metoda lokalizacji uszkodzeń bazująca na lokalnych filtrach modalnych została opracowana [2], jako rozszerzenie metody wykrywania uszkodzeń w oparciu o filtrację przestrzenną [3]. Metoda lokalnych filtrów modalnych poprawnie lokalizuje uszkodzenie na obiekcie. Jednakże istnieją przypadki, gdy w obiekcie rozwija się więcej niż jedno uszkodzenie. W niniejszym artykule autor postara się odpowiedzieć na pytanie, czy wspomniana metoda okaże się skuteczna w takiej sytuacji. W symulacjach kolejno wprowadzano uszkodzenia o różnym wymiarze i lokalizacji i sprawdzano czy są one wykrywalne oraz czy ich wzrastająca liczba wpływa na dokładność lokalizacji.

Słowa kluczowe: filtracja modalna, wykrywanie uszkodzeń, lokalizacja uszkodzeń.

1. INTRODUCTION

Within the low-frequency vibration based group of methods for damage localization one can notice that most of them are based on analysis of mode shapes changes. There are many methods that uses this modal parameter. They can be divided into three groups:
- tests of correlations between mode shapes in undamaged and current state (CoMAC),
- analysis of mode shapes curvature,
- analysis of mode shapes deformation energy.

The MAC (Modal Assurance Criterion) coefficient is defined as a scalar product of two modal vectors [4], from which the first is identified for an undamaged system, and the second is a mode shape for the object with damage. If the MAC coefficient is less than one, there is a change in the vibration mode. The MAC coefficient may be calculated both for one coordinate and for a certain area. It was applied to damage detection in the work [5]. For one selected coordinate, it is called *CoMAC* (*Coordinate MAC*). Designating it can additionally define in which area the damage is located. In practice, however, the method is not very sensitive and does not allow damage to be detected in the initial phases of development.

A development of these methods is the analysis of changes in mode shape together with changes in the mode participation factor. This method was described in the paper [6]. The authors divided the analytical model into sub-systems and analysed changes in higher mode shape in successive sub-systems. This method allows the localisation of damage, because higher mode shapes are changed only for sub-systems containing damage.

A slightly different use of mode shapes for damage detection was proposed by Ettouney [7]. He calculated the stiffness matrix or compliance matrix on the basis of knowledge of the modal model of undamaged and damaged objects. Changes in the calculated matrices indicated the presence of damage and its location.

In the place of modal vectors, a change in the curvature defined as the derivative or the second derivative of a modal vector are often analysed. This is more sensitive to changes than the mode itself. In particular, this concerns damage to objects which changes mode shapes locally. This method suffers from a relatively large error in cases where the number of measurement points is not sufficiently large to designate the following derivative of vibration modes with suitable accuracy. The derivative is calculated in points by linear approximation passing through the successive points or also through polynomial approximation of the deformation curve and derivative designation analytically. This second method is considerably less sensitive to measurement errors, however, it flattens the shape of modes, which may be a cause of damage being undetected. The effectiveness of applying these methods also includes the localisation of damage.

The first example of the application of mode shape curvature to NDT is the work of Maeck and DeRoeck from 1999 [8]. This method uses dependencies between the beam's bending stiffness and the bending moment divided by a suitable curve, being a second derivative of beam deformation. Changes in the stiffness matrix calculated on the basis of this dependency enable the detection of damage. This method was verified by the authors with the use of data from research conducted on the Z24 bridge in Switzerland.

The next application of modal curvature is the method described in the article [9]. The authors calculated the damage index as a relation of the modal curvature calculated for the damaged object to the analogical curvature of the undamaged object raised to a square. The curvature was counted as a second mode shape derivative. In the work, particular emphasis was placed on the influence of measurement errors on the accuracy of method. They showed that the higher mode shape derivatives are more sensitive to the presence of damage, but also cause multiplications of measurement noise and, due to this, their usefulness is doubtful. In their next publications [10], the same authors proposed a solution to the problem of increasing significance of measurement noise during analysis of changes in curvature. They presented another way of mode shape analysis, which had a high level of sensitivity to damage and low one to measurement error. As a symptom of damage, they proposed a change in the mode shape slope raised to a square. This slope was counted as the first mode shape derivative.

The damage index method from the work [9] was extended by Kim and others [11]. The novelty is based on the application of the method for objects where reference data (without damage) was not available. The authors showed a way to calculate the modal curvature of the object before damage on the basis of data coming only from measurements on the damaged object. In this method it was necessary to use the updated finite element model.

A completely different approach for damage localisation on the basis of mode shapes was presented by Rucka and Wilde [12]. They analysed the mode shape of beam-like and plate-like objects in search of cracks. The tool which was used for this aim was the wavelet transform. The discovered discontinuities, where the geometry of the object was known, were the symptom of damage. This method requires very dense networks of measurement points. Its application to the mode shapes obtained through laser vibrometer measurements are presented in the work [13]. The main advantage of the approach is the fact, that it is a baseline free method. The accuracy of the method was further increased in [14]

The most precise method based on modal vectors is presented in the work [15]. This method consists of comparing the deformation energy of vibration modes in systems without damage and systems with damage. In the described method a finite element model of the construction can be considered as a system without damage. In order to designate the SER_{ij} energy coefficient, the i-th vibration mode for the j-th element, one should possess the following data: mass normalized mode shape ϕ_i, natural frequency ω_i, the global stiffness matrix of the finite element model K, as well as the stiffness matrix for the j-th finite element k_j:

$$SER_{ij} = \frac{\phi_i^T k_j \phi_i}{\phi_i^T K \phi_i} = \frac{\phi_i^T k_j \phi_i}{\omega_i^2} \tag{1}$$

The coefficient β_{ij} is named by the authors as a damage coefficient and is calculated as a difference between energy coefficient designated for undamaged and damaged structure. As simulation and experimental tests showed, this method is sensitive even to small stiffness changes in the construction (about 5%).

A similar approach can be found in the work [16]. The authors also calculated the damage indicator on the basis of the deformation energy of mode shapes. However, in this case, it was defined a little differently:

$$f_{ij} = \int_{a_{j-1}}^{a_j} \left(\frac{d^2\phi_i}{dx^2} \right)^2 dx \Big/ \int_0^L \left(\frac{d^2\phi_i}{dx^2} \right)^2 dx \tag{2}$$

where: i – mode shape number,
j – element number,
L – length of section on which the mode shape curvature is calculated,
x – position on the section L,
a – integration limit.

The damage indicator was calculated as a relation of the sums f_{ij} along all mode shapes for the damaged object to analogical sums of the undamaged object.
Carrasco et al. [17] also applied mode shape deformation energy for damage detection and localisation. Their approach consists of dividing the tested object into sub-systems and the calculation of deformation energy separately for each sub-system. Changes in deformation energy in successive sub-systems allowed the authors to locate damage. Additionally, the authors showed that there is a close dependency between the size of the damage and the size of the change.
The mode shape deformation energy for damage detection and localisation was also applied by Choi and Stubbs [18]. Their work concerned damage detection in a two-dimensional element with the use of classic plate theory. As an example, they applied a finite element model to a rectangular plate.
Recently one more method of damage localization based on mode shapes has been developed [2, 19]. It uses a modal filtration output characteristics to generate diagnosis.

2. DAMAGE DETECTION WITH USE OF MODAL FILTRATION

The method was originally developed for damage detection [3]. But to present it properly it is first necessary to remind a few words about modal filtration.
The modal filter is a tool to extract the modal coordinates of each individual mode from the system outputs by mapping the response vector from the physical space to the modal space [20]. It was first introduced by Baruh and Meirovitch in 1982 [21] to overcome the spill-over problem within control of distributed parameter systems. Spill-over is a phenomenon in which the energy addressed to the controlled mode is pumped into the uncontrolled modes. To construct the modal filter a new modal parameter was introduced: reciprocal modal vectors. They should be orthogonal with respect to all the modal vectors except the one to which the filter is tuned, and, thanks to that, are applied to the decomposition of the system responses to the modal coordinates η_r.

$$\eta_r(\omega)=\psi_r^{\,T}\cdot\{x(\omega)\}= \\ =\left(\frac{\{\phi_r\}^T}{j\omega-\lambda_r}+\{\psi_r\}^T\{\phi_r^*\}\frac{\{\phi_r^*\}^T}{j\omega-\lambda_r^*}\right)\cdot\{f(\omega)\} \qquad (3)$$

where: $x(\omega)$ – vector of system responses.

Now, scaling the modal coordinates η_r by the known input, it is possible to determine the FRF with all peaks, except r – th, filtered out. The way of using modal filtering for structural health monitoring is presented in Figure 1.

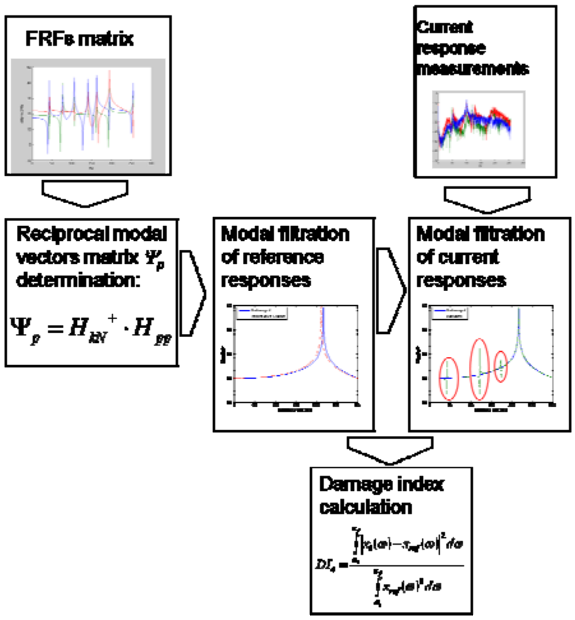


Fig. 1. Procedure for damage detection with use of modal filtration

The frequency response function of an object filtered with a modal filter has only one peak corresponding to the natural frequency to which the filter is tuned. When a local change occurs in the object – in stiffness or in mass (this mainly happens when damage in the object arises), the filter stops working and, on the output characteristic, other peaks start to appear, corresponding to other imperfectly filtered natural frequencies. On the other hand, a global change in entire stiffness or mass matrix (due to changes in ambient temperature or humidity) does not corrupt the filter and the filtered characteristic still has one peak, although it is slightly moved in the frequency domain

3. DAMAGE LOCALIZATION WITH USE OF MODAL FILTRATION

The idea to extend the method by adding damage localization [2, 19] is based on the fact that damage, in most cases, only disturbs the mode shapes locally. It is then possible to divide an object into areas measured with the use of several sensors and build separate local modal filters for data coming only from these sensors. In areas without damage, the shape of modes does not change and the modal filter keeps working – there are no additional peaks on the filter output. When a group of sensors placed near the damage is considered, mode shape is disturbed locally due to damage and the modal filter does not

perfectly filter the characteristics measured by these sensors.
It has been said [20] that the minimum number of sensors required to build an effective modal filter is equal to or greater than the number of modes in the frequency range of interest. Thus, by limiting the frequency range of analysis to the first two modes, it would be theoretically possible to construct modal filters for object areas measured with only 2 sensors. This would significantly increase the damage location accuracy. In practice, however, to construct the modal filter for two modes, data from at least 4 sensors has to be used. The accuracy of damage localization depends on measuring net density. The graphical presentation of this idea is presented in Figure 2.

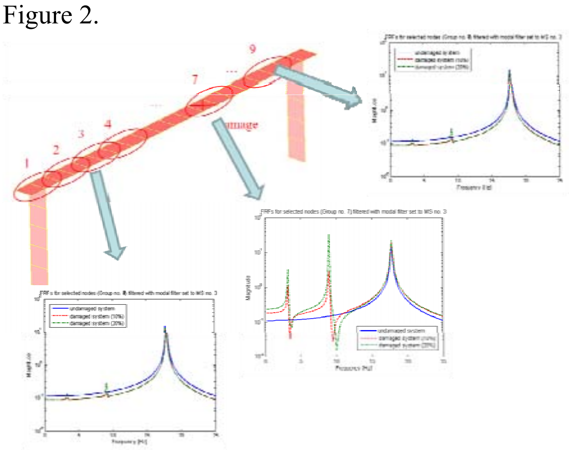


Fig. 2. Scheme of the proposed method of damage localization

To make localization easier, it was decided to use the damage index DI provided by Equation (4):

$$DI = \frac{\int_{\omega_s}^{\omega_f} \left| x_i(\omega) - x_{ref}(\omega) \right|^2 d\omega}{\int_{\omega_s}^{\omega_f} x_{ref}(\omega)^2 \, d\omega} \qquad (4)$$

where: ω_s – starting frequency of the analyzed band,
ω_f – closing frequency of the analyzed band
x_i – characteristic in the current state
x_{ref} – characteristic in the reference state

The DI is calculated only for the frequency regions which directly surround the Natural frequencies of the object, except the one to which the modal filter is tuned. The width of the consecutive frequency intervals was assumed to amount to 20 % of the corresponding natural frequency. For practical applications it would be useful to set the threshold for the difference between damage index value for the regions (groups of sensors) with and without damage. Such a threshold would make it possible to detect damaged region more easily and even introduce some automation of diagnostic procedure. However, the threshold needs to be assessed individually depending on the object type and minimum damage size to be detected.

4. SIMULATION OF MULTIPLE DAMAGE LOCALIZATION

So far, the method was tested on simulation and experimental data, but always verification was carried out with one damage present in the object. Now the question arises, how the method will behave with multiple damage. The basic assumptions of the method of damage localization is based on the work of Pandey et al. [22], where it is shown that a local change of stiffness locally changes the shape of the mode of vibrations. This also applies to multiple damages. Every damage disturbs the mode shape separately in the area of its occurrence. If the distance between the damages is bigger than size of the sensor group used for local modal filter identification (the damages are in different groups) then it is possible to identify them. Of course every damage apart from its local impact on the shape of mode has also its global influence. The bigger number of damages is present in the object, the greater this global influence is, with respect to the local ones and proper localization of multiple damages will be more difficult.
To confirm the hypothesis that it is possible to localize multiple damages a finite element model was prepared. This was a steel supported beam of length 10 m with the cross-section dimensions 0.6 x 0.1 m and consisted of 600 plate elements (quad4), with the size of each element 0.1 x 0.1 m. Such a dense mesh was used to allow for multiple damage detection testing. Next, the eigenvalue problem was solved for the model without damage to obtain its modal model parameters. As damage, a beam cracks were modeled as node disconnectivity. The depth of the crack amounted to 10% of the entire beam cross-section area. The cracks were added consecutively (one by another in following simulations) . The finite element model used for simulations is presented in Figure 3.

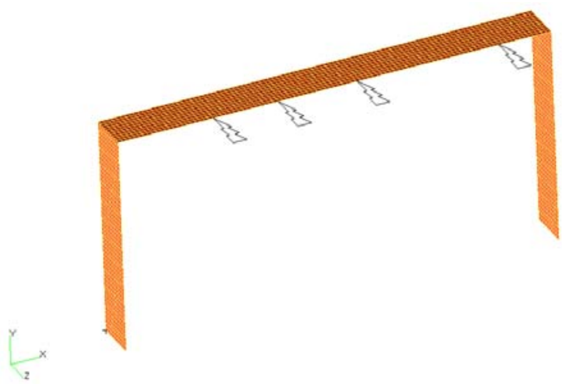


Fig. 3. Finite Element model used for simulation

The cracks were introduced in the upper bar of the frame in the distance 2.2, 3.4, 5.4 and 9 m from the left end of the bar. For the damage localization 200 nodes (virtual sensors) were selected. They were evenly spaced every 0.1 m in two lines on both edges along the length of the bar. The virtual sensors were divided into 49 groups (6 sensors in each with two sensors overlapping). The picture showing the sensor placement and group definition is shown in Figure 4.

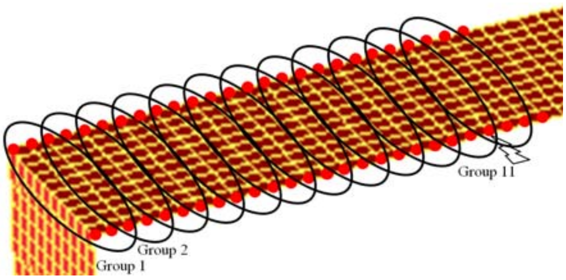
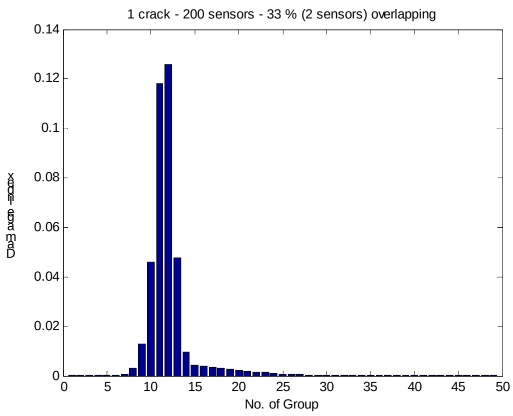


Fig. 4. Groups of sensors definition

With such a group definition, the cracks were placed in 11th, 17th, 27th, and 45th group respectively. Results of simulation are placed in Figure 5.

As it is clearly visible on the bar plots all four cracks were detected and localized properly. It is however noticeable, that consecutive cracks (of the same depth) give different values of DI. It is caused by the fact, that for the DI calculation the local modal filters tuned to the 3rd bending mode were selected. And according to the paper [3] the best detectable (biggest DI value) are the cracks which are localized in the most deformed area of selected mode.



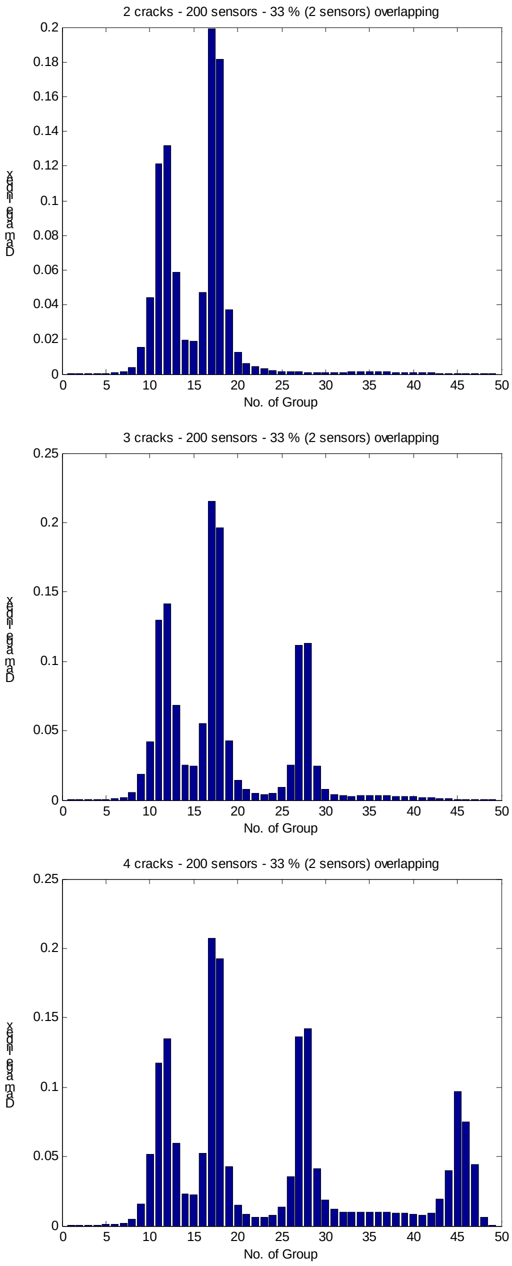


Fig. 5. Results of multiple crack localization

5. CONCLUSIONS

The paper presents the simulation verification of the damage localization procedure in case of many damages present in the object at the same time. The presented investigations can be summarized by the following conclusions:

- the method is able to detect multiple damages,
- level of DI for is different for consecutive damages and it depends on its localization with respect to the shape of the mode to which the local filters are tuned to,
- the method is able to distinguish between neighboring damages if the distance between

them is bigger than size of the group of sensors for which the local modal filter is identified.

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