APPLICATION OF ARMA MODELLING AND ALPHA-STABLE DISTRIBUTION FOR LOCAL DAMAGE DETECTION IN BEARINGS

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Summary

In this paper a novel method for informative frequency band selection is presented. It is suitable for a vibration signal from a damaged rotating machine which is consisted of a pulse train, but it might be contaminated by other vibrations, often with higher energy. We first decompose the signal into simpler sub-signals and analyze those sub-signals using statistical tools, i.e. autoregressive moving average modelling and fitting of the α -stable distribution. The choice of this distribution is motivated by its excellent ability of modeling heavy-tailed data, i.e impulsive data. We illustrate the proposed methodology by analysis of real vibration signals from heavy-duty rotating machinery. The results prove that this statistical analysis is very efficient in informative frequency band selection in presence of high-energy contamination.

Keywords: local damage detection, informative frequency band selection, time series modeling, heavy-tailed distribution

ZASTOSOWANIE MODELU ARMA I ROZKŁADU ALFA-STABILNEGO DO DETEKCJI USZKODZEŃ LOKALNYCH W ŁOŻYSKACH

Streszczenie

W artykule zaprezentowano nową metodę selekcji informacyjnego pasma częstotliwościowego. Jest ona odpowiednia dla sygnałów drganiowych z maszyny uszkodzonej zawierających impulsy, nawet kiedy są one niewidoczne w dziedzinie czasu, tzn. kiedy wysokoenergetyczne drgania innych elementów zakłócają sygnał informacyjny. Pierwszym krokiem zaproponowanej metody jest dekompozycja sygnału na składowe o prostszej strukturze i ich analiza za pomocą narzędzi statystycznych, tj. modelu ARMA i rozkładu alfa-stabilnego. Wybór tego rozkładu jest umotywowany zdolnością modelowania danych ciężko ogonowych, tzn. sygnałów, w których występują impulsy. Metodę zilustrowano analizą rzeczywistych sygnałów z drganiowych maszyn górniczych. Potwierdzono efektywność zaproponowanej metody statystycznej w kontekście selekcji informacyjnego pasma częstotliwościowego w obecności wysokoenergetycznych zakłóceń.

Słowa kluczowe: wykrywanie uszkodzeń lokalnych, wybór pasma informacyjnego, modelowanie szeregów czasowych, rozkłady ciężko-ogonowe

1. INTRODUCTION

Diagnostics of belt conveyor drive systems, which is an example of a heavy-duty machinery, is a very challenging problem due to its complex structure [1]. One of the most popular methods of damage detection that do not require visual inspection of the diagnosed machine relies on processing of a vibration signal acquired on the housing. Since such signal contains a lot of different components, extraction of the part which is informative from diagnostic point of view is the key issue. In other words, the signal has to be decomposed into the informative and non-informative part. In the literature one can find several methods that are based on the fact, that different frequency bands of the considered signal might carry different level of diagnostic information. Thus it is of high importance to divide the frequency spectrum of the signal into sub-bands to assess each one and quantify its diagnostic informativeness. The informativeness might be quantified by using many different approaches. Since a local damage of the machine results in a pulse train in time domain, thus it is revealed by a set of wideband excitations in the time-frequency domain. In the case of constant rotational speed (or when angular resampling make distances between pulses constant) the excitations constitute a periodic pulse train in a frequency band where they appear, thus not only impulsiveness, but also periodicity of impulses might be taken into consideration for assessment and quantification of

informativeness. In the literature one can find methods that are based on the periodic nature of the pulse train, e.g. the protrugram [2], statistic based on the local maxima method [3,4]. There are also many previous works that incorporate the cyclic nature of a vibration signal from rotating machinery [5-10]. If the pulses might not be periodic or angular resampling does not give good results (or it just cannot be applied), then some methods that exploit only impulsiveness might be used, e.g. kurtosis [11-13], sparsity [14-16], measures based on moments, quantiles or cumulative distribution function [3,17] and other statistical criteria for informative band selection [18-21]. In this paper we follow the second approach, but with indirect application of the novel quantitative measure of informativeness. The indirection means that the measure is not applied to the raw sub-signals that represent energy flow in particular sub-bands, but to preprocessed sub-signals. Such preprocessing is required by the measure we propose, namely we decorrelate sub-signals to obtain data to which we can properly fit the stability parameter of the α-stable distribution, i.e. the α parameter. One of the motivations of the preprocessing step is related to appropriate assessment and quantification of certain types of sub-signals. During the process of dividing the raw signal into sub-signals it can be noticed that some sub-signals might reveal a strange behavior, e.g. due to specific windowing procedure while sub-signals are obtained by the short-time Fourier transform (STFT). Therefore, the decorrelation procedure might decrease influence of data correlation on the measure of impulsiveness and increase influence of impulses related to the local damage.

The paper is organized as follows. In Section 2 we present methodology that incorporates short time Fourier transform, sub-signals extraction, modeling of sub-signals, statistical analysis of residuals and selection of the informative frequency band. In Section 3 we present analysis of a real data set which illustrates properties of the proposed methodology. The last section contains conclusion.

2. METHODOLOGY

In this section we present the algorithm that is used for selection of the informative band (Fig.1). After obtaining the raw signal, we first transform it into the time-frequency map through the short-time Fourier transform (STFT). The STFT is defined as [22]:

$$STFT(t,f) = \int_{-\infty}^{\infty} X(\tau)w(t-\tau)e^{2j\pi f\tau}d\tau \qquad (1)$$

where $w(t-\tau)$ is the shifted window and $X(\tau)$ is the input signal. The discrete version of equation (1) for

observations $X_1, X_2, ..., X_N$, time point $t \in T$ and frequency $f \in F$ is defined as follows:

$$STFT(t,f) = \sum_{k=0}^{N-1} X_k w(t-k)e^{2j\pi fk/N} \qquad (2)$$

In our analysis we use the Kaiser window. In the proposed procedure each sub-signal is a time series corresponding to a narrow frequency band that arises after the mentioned time-frequency decomposition. However, since the STFT matrix is complex, absolute value needs to be taken to obtain the spectrogram. Moreover in our analysis we examine the increments of the sub-signals. In the next step we are going to determine orders and coefficients of the ARMA model through prediction error minimization algorithm.

The time series X_t is said to be an autoregressive moving-average process of order (p,q) ($ARMA(p,q)$) if $\{X_t\}$ is stationary and if for every t the following equation is satisfied [23]:

$$\varphi(B)X_t = \theta(B)\varepsilon_t \qquad (3)$$

where B is backwards-shift operator: $BX_t=X_{t-1}$ and

$$\varphi(B)=1- \varphi_1 B^1 -...- \varphi_p B^p$$
$$\theta(B)=1 + \theta_1 B^1 + \theta_q B^q \qquad (4)$$

It is well-known how to estimate parameters of ARMA model - it might be done through Yule-Walker equations, Maximum Likelihood Estimator or prediction error minimization algorithm [23]. Last one is used in this work.

Another important issue related to the ARMA time series model are information criteria [24]. By choosing many possible orders of ARMA for both AR and MA parts (by ACF and PACF), and then applying an information criterion, e.g. the AIC criterion, one can choose best-fit model [23]. The next step involves statistical analysis of residuals of aforementioned models. Residuals are calculated by following equation:

$$\varepsilon_t = \frac{\varphi(B)}{\theta(B)} X_t \qquad (5)$$

We propose to fit the is α-stable distribution to the residuals, since in sub-signals we expect a set of impulses (in statistical meaning − a set of extreme values) in a damaged bearing case and the is α-stable distribution can perfectly exhibit such behavior. This distribution has been already used for diagnostics, see [25]. A random variable X is α-stable distributed if its characteristic function is as follows [26]:

$$Ee^{jtX} = \begin{cases} \exp(-\sigma^\alpha |t|^\alpha (1-\beta \text{sgn}(t)\tan(\frac{\pi\alpha}{2}))+i\mu t) & \alpha \neq 1 \\ \exp(-\sigma |t|(1+i\beta \text{sgn}(t)\frac{2}{\pi})+i\mu t) & \alpha=1 \end{cases} \text{, (6)}$$

where α is stability parameter, β is asymmetry parameter, σ is scale parameter and μ is location parameter. The α-stable distributed random variable has 'heavy' tails, i.e. its cumulative probability density function decays with power law. Therefore, there is a high probability of the variable having extreme values, which is useful in modeling signal with impulses. Therefore if we expect signal to come from damaged bearing, we shall expect high impulses on the left and right (tails). One of them method used to estimate parameters of the α-stable distribution is called the McCulloch method and uses quantiles of the distribution [27].

On this basis we define a new selector for the informative band as:

$$Sel_\alpha = 2-\alpha \qquad (7)$$

with $0<\alpha\leq2$. This formula is motivated by the fact that for non-heavy tailed data (i.e. for healthy or non-informative) the α parameter should be close to 2 and for heavy-tailed – lower than 2.
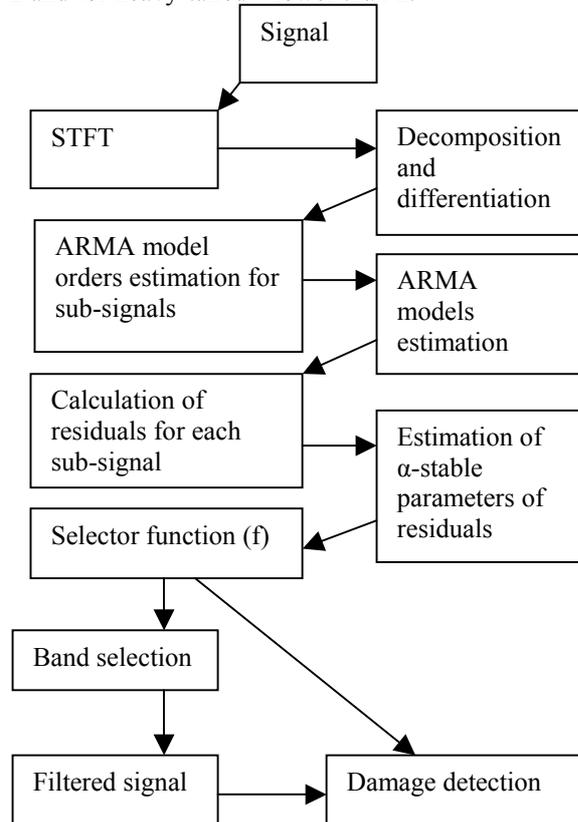
Fig. 1 Block scheme of analysis

Thus, the selector is close to 0 in healthy case and significantly higher than 0 for a data with set of impulses, which is consistent with previously used criteria, for example, kurtosis. Mentioned selector is higher for signals containing impulses (α is lower) and it is invariant to energy contained in the considered narrowband frequency bin. We also refer to [28] for detailed analysis of the α-stable distribution with α parameter close to 2.

Therefore, above methodology might be helpful in selection of informative frequency band.

3. REAL DATA ANALYSIS

In this section we present real data analysis performed for validation of the proposed method of informative frequency band selection described in the Section 2. The investigated signals represent vibrations of a bearing which is a part of a belt conveyor driving unit. The unit is consisted of a motor, two-stage gearbox and drive pulley. The signals were acquired using an accelerometer located on the bearing's housing in horizontal direction. The machine was normally loaded and the rotational speed was approximately constant during the data acquisition. We analyze two signals – one related to a healthy bearing and one related to a bearing with local damage of the outer race. Sampling frequency in both signals is 19.2 kHz and length of both of them is 2.5 s. These signals were analyzed in several previous works, where one can find detailed description of the machine. We just recall that the theoretical characteristic frequencies are: FTF (fundamental train frequency) – 0.51 Hz, BSF (ball spin frequency) – 4.45 Hz, BFF (ball fault frequency) – 8.9 Hz, BPFO (ball passing frequency outer race) – 12.34 Hz and BPFI (ball passing frequency inner race) – 16.06 Hz. Fig. 2 presents time series of the raw vibration signals.

Fig. 3 and 4 shows the corresponding spectrograms.
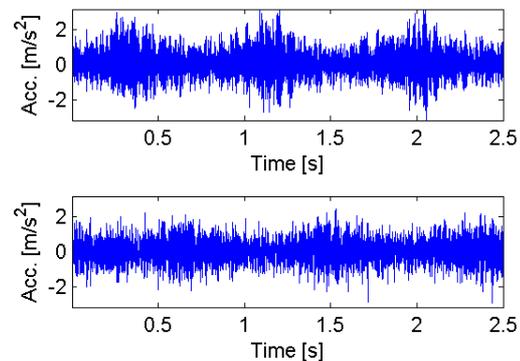
Fig.2 Healthy (top panel) and damaged (bottom panel) bearings raw vibration signal

Parameters of the spectrogram are as following: Kaiser window of length 125, the number of overlapping samples is equal to 110 and the number of FFT points is 512. In both cases high-energy, low-frequency contamination from the gearbox is present (below 1 kHz) as well as high-frequency, low-energy noise above 7 kHz. Middle band from 1 to 7 kHz contains information about cyclic impulses which are related to the local damage.
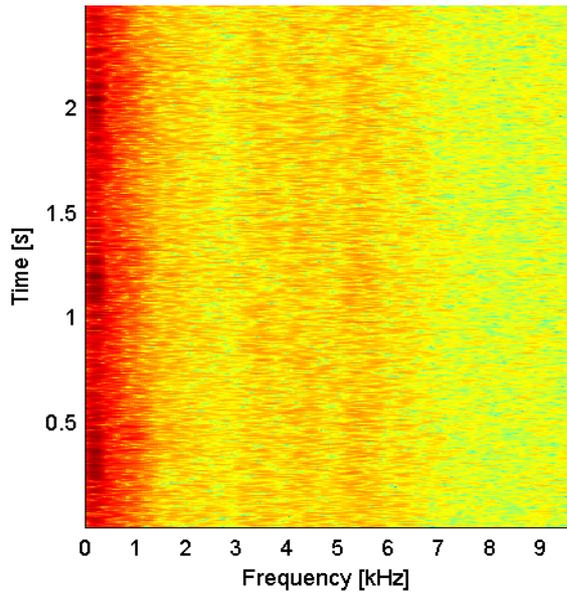
Fig.3 Spectrogram of vibration signal from healthy bearing
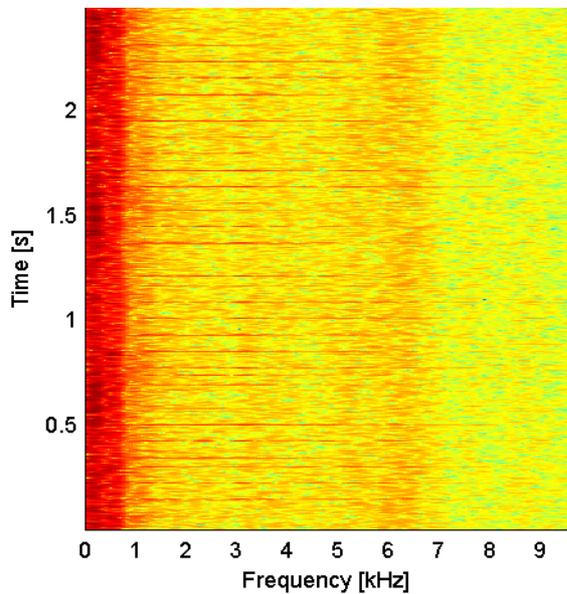
Fig.4 Spectrogram of vibration signal from damaged bearing

Fig.5 Orders of AR part of estimated ARMA
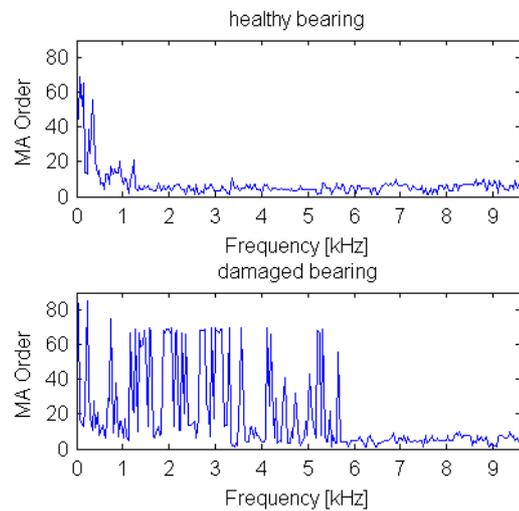
Fig.6 Orders of MA part of estimated ARMA model

Figures 5 and 6 represent orders of coefficients of AR and MA parts, respectively, chosen through the Akaike Information Criterion. As it can be seen orders of AR part for damaged bearing are significantly higher in band 1kHz-6kHz. Observing Figure 6 one can notice that orders of MA part behave in similar way. Observing both, AR and MA part one can notice that orders for damaged bearing are significantly higher for MA part in informative band. It is due to the series of high energy impulses which increase the order of MA part.

Figures 7-12 present exemplary residuals of the suitable ARMA model from 3 narrowband frequency bins: low frequency, middle-frequency and high-frequency. Each frequency bin is represented by 2 sub-signal residuals – one from a healthy bearing and one from the damaged one.

It can be noticed that sub-signal residuals for low- and high-energy frequency bins in both cases are free of impulses related to damage, but they do not seem to be normally distributed. Thus, the stability parameter of the α-stable distribution should be close to 2 which results in close to 0 value of the selector defined in Eq. (7). Also sub-signal residuals for healthy case from the middle-frequency bin does not reveal pulses characteristic for low parameter of stability in the α-stable distribution. The only sub-signal residual which is consisted of impulses represents the middle-frequency bin and the damaged bearing. The value of the selector should be significantly higher than 0 therein.
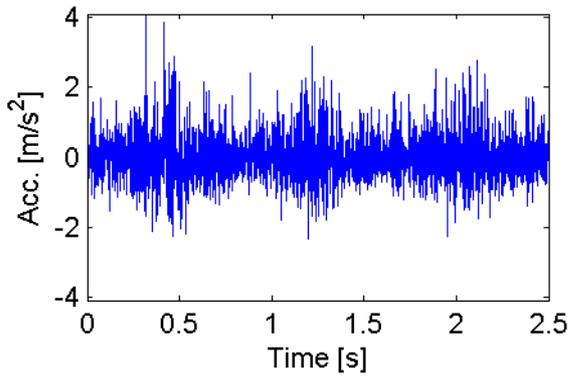
Fig.7 Sub-signal residuals for healthy bearing corresponding to 900 Hz frequency bin
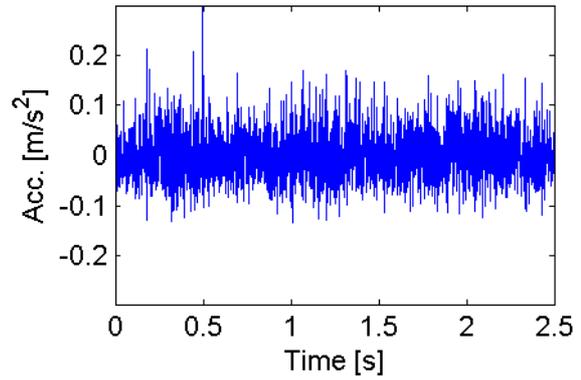
Fig.8 Sub-signal residuals for damaged bearing corresponding to 900 Hz frequency bin
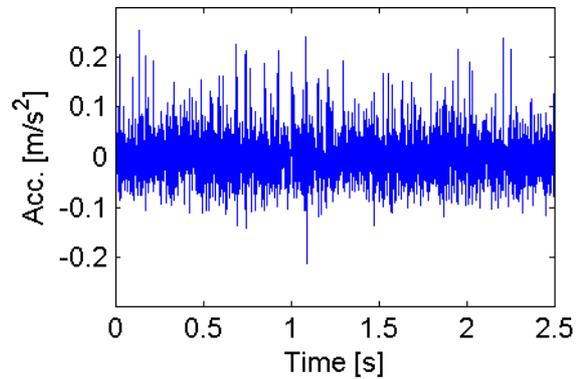
Fig.9 Sub-signal residuals for healthy bearing corresponding to 3375 Hz frequency bin

Fig.10 Sub-signal residuals for damaged bearing corresponding to 3375 Hz frequency bin

Fig.11 Sub-signal residuals for healthy bearing corresponding to 7462,5 Hz frequency bin

Fig.12 Sub-signal residuals for damaged bearing corresponding to 7462,5 Hz frequency bin
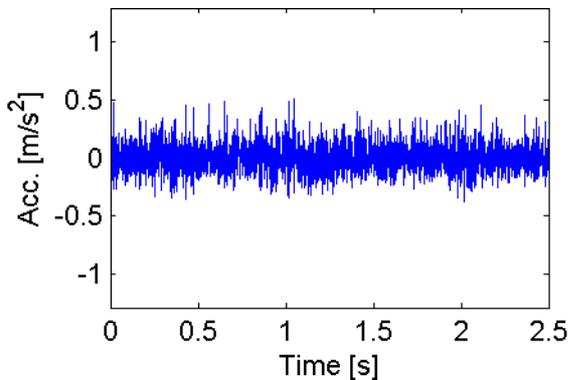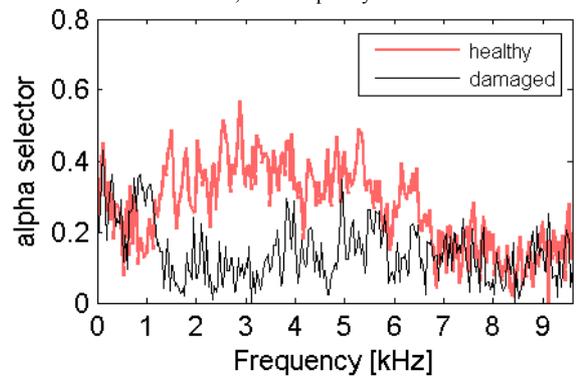
Fig.13 Selector values for all range of frequencies
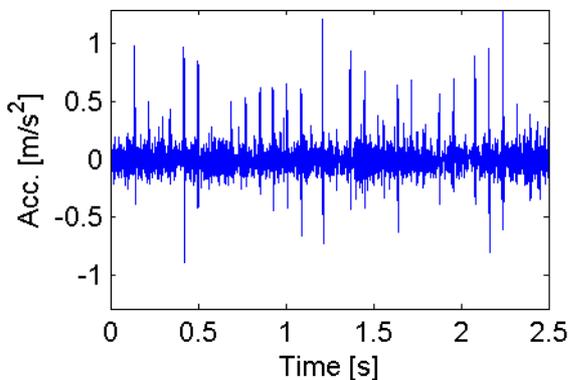
Figure 13 presents values of selectors for all range of frequencies for healthy and damaged case, respectively. It can be easily noticed that for the frequency band between 1 and 7 kHz values of the selector for damaged bearing signal residuals are significantly higher. It proves effectiveness of our algorithm in automatic selection of the informative frequency band.
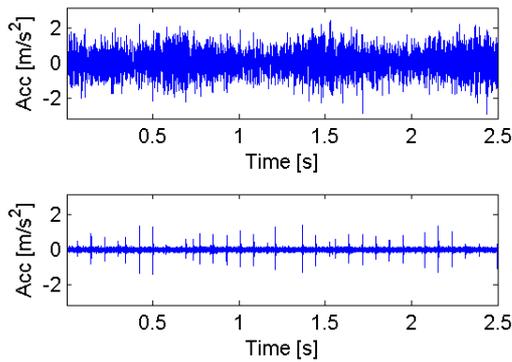
Fig.14 Damaged signal after filtration through band-pass filter frequency 1-7kHz. Top panel – signal from damaged bearing before filtration, bottom panel – after filtration

In Figure 14 signal from bearing with damaged outer ring is seen (top panel). After band-pass filtration applied with lower frequency equal to 1kHz and upper frequency equal to 7kHz, we obtained signal with clear visible impulses. Therefore, our method have been proven to be effective.

4. CONCLUSION

In the paper a novel criterion for informative frequency band selection is proposed. It is based on the fundamental property of a vibration signal from a damaged machine, i.e. presence of impulses in a certain frequency band. As an indicator of impulsiveness we used the stability parameter of the α-stable distribution. The choice of this parameter is motivated by the fact that it is close to a certain constant for a sub-signals that represent healthy machine or the non-informative part of the signal from a damaged machine and it is significantly lower for frequency bands containing impulses. Since the sub-signals are correlated due to the high overlapping in the STFT, we first model the sub-signals using the ARMA time series model of optimal order to improve efficiency of the selector. We illustrated the efficiency of the proposed method by analysis of a vibration signals from a heavy-duty machinery – drive pulley bearing from a belt conveyor driving unit. We proved that the novel selector appropriately finds the informative frequency band and the filtered signal contains a set of impulses that was invisible in the raw vibration signal.

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