PIEZOELECTRIC SHUNT DAMPING FOR ALPINE SKI TRACTION IMPROVEMENT

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Summary

The paper investigates the problem of improving driveability of alpine skis. That properties depend largely on the vibration behavior of a ski under different snow conditions. There is a broad range of methods to deal with vibration damping, both active and passive, that have been described in the literature. In this paper the passive approaches were taken into consideration. It is due to the specific nature of the sport where use of external sources of energy may be impractical.

Passive shunt damping system has been designed for a commercially available model of an alpine ski. The performance of the designed system has been evaluated on a numerical model of the ski. Piezoelectric patches have been included in the model by means of a generalized impedance that complies both the mechanical properties and shunted circuit electrical properties. The fraction of strain energy stored in a piezoelectric element at particular vibration modes were used to evaluate the maximum damping coefficient for first three resonant modes that have the largest influence on the ski properties. The damping system is based on piezoelectric patches that can be easily embedded in the laminated structure of a ski.

Keywords: vibration damping, shunt damping, piezoelectric patches

SYSTEM TŁUMIENIA DRGAŃ Z WYKORZYSTANIEM MATERIAŁÓW PIEZOELEKTRYCZNYCH DLA POPRAWY WŁASNOŚCI TRAKCYJNYCH NART ZJAZDOWYCH

Streszczenie

W artykule przeprowadzono analizę możliwości poprawy własności jezdnych nart zjazdowych z wykorzystaniem materiałów piezoelektrycznych. Właściwości jezdne narty zależą w dużej mierze od ich własności dynamicznych i drgań wzbudzanych podczas jazdy w różnych warunkach śniegowych. Do tej pory podejmowano próby wykorzystania zarówno metod pasywnych jak i aktywnych dla poprawy trakcji. W artykule wzięto pod uwagę tylko metody pasywne, odrzucając metody aktywne ze względu na konieczność użycia zewnętrznego źródła zasilania co uznano za niepraktyczne w omawianym zastosowaniu.

W artykule przedstawiono pasywny układ tłumienia drgań typu „shunt damping”, który został zaprojektowany dla wybranego komercyjnego modelu nart zjazdowych. Działanie zaproponowanego układu przeanalizowano na uprzednio z walidowanym modelu numerycznym.
Wpływ elementów piezoelektrycznych oraz bocznikującego układu elektrycznego uwzględniono stosując metodę impedancyjną oraz posługując się uogólnionym współczynnikiem sprzężenia elektromechanicznego. Ułamek energii odkształcenia sprężystego danej postaci zawarty w materiale piezoelektryka został wykorzystany do wyliczenia możliwych do uzyskania maksymalnych współczynników tłumienia modalnego dla pierwszych trzech postaci, najbardziej znaczących z punktu widzenia własności jezdnych. System tłumienia drgań został tak zaprojektowany, aby mógł być w prosty sposób zaimplementowany w konstrukcji nart zjazdowych.

Słowa kluczowe: tłumienie drgań, tłumienie drgań z wykorzystaniem materiałów piezoelektrycznych, elementy piezoelektryczne

1. INTRODUCTION

There is an observable trend towards the smart sports equipment. One of the most spectacular and technologically challenging sports is alpine skiing. It is known that the traction properties of alpine skis strongly depend on their stiffness. The layered construction of a ski allows to shape the longitudinal and lateral stiffness maintaining its low weight. However, due to diverse snow conditions, some dose of structural adaptivity is required to keep good traction properties anytime. It has been shown that damping of the first three vibration modes predominantly influence the ski traction properties [1].

Piezoelectric elements in industry have been intensively studied in the last few decades. A number of passive and active techniques of vibration damping makes application of piezoelectrics more available also in sports like skiing. So far three equipment producers have used piezoelectrics in alpine skiing, however, very little attention has been gained by scientific community. The known embodiments [1, 2, 3, 4, 5] assume three distinct locations for piezo patches: under the binding and at the ski tip and heel depends on the effect to be achieved. The piezoelectrics are implemented either in the form of patches or amplified piezoelectric actuators (APA). The patches can be combined together [1] to form large area dampers. Majority of works use active control scheme where the piezoelectric elements are externally powered based on the measured ski vibrations. Although the mass of the actuator itself is not significant the mass of the additional structural elements of APA, control electronics and power sources are of great concern. For the known solutions [2] the mass of abovementioned equipment may be as large as twice the mass of the ski. The change in system dynamics caused by added mass and its distribution is not negligible too. The problem of high frequency vibrations which appears under operational conditions has also been reported [5]. However, the problem is rather related with the particular implementation than with the method itself. Beside the active control the switching control methods have also been investigated. Synchronized Switch Damping on Inductor (SSDI) has been successfully implemented [3] allowing decrease in torsional ski vibrations. Finally, the passive means of vibration reduction with the use of piezoelectric elements have also been implemented [1], although in the form of resistive shunt circuit of limited performance (the effect comparable to light visco-elastic damping [6]).

Widely used resonant shunt circuits that proofed reasonable tradeoff between performance and limitations that results from its passivity has not been investigated so far. The paper presents the study of the influence on the first vibration modes achievable with RL circuit.

2. EXPERIMENTAL SETUP

The study has been performed on a commercial version of the Rossignol 9s Ti Oversize slalom ski made for the season 2005/2006. The ski was analyzed without binding. The layered structure of the ski is shown in Fig. 1. For modal testing, the ski was freely suspended on elastic cords to simulate the free-free conditions. Excitation was supplied by the *TMS K2007E01* electromagnetic shaker attached to the ski and driven by the white noise signal. Vibration velocity responses were measured in a non-contact manner in 335 points by the *Polytec PSV-400* Scanning Laser Doppler Vibrometer (SLDV). Polytec PSV-400. The Frequency Response Functions (Fig. 3) were calculated from the experimental input and output data using the *Polytec PSV* software. Experimental setup is shown in Fig. 2.

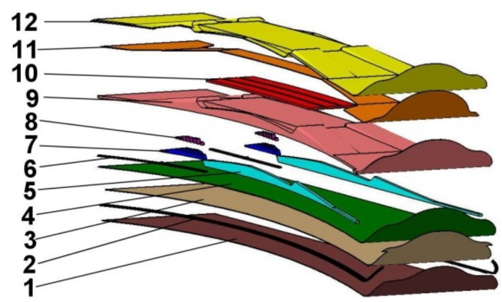


Fig. 1. The exploded view of the model of a ski: 1) base, 2) edge, 3) fiberglass, 4) aluminum, 5-8) composite sidewalls, 9) nomex core, 10) elastomer core, 11) aluminum, 12) top sheet

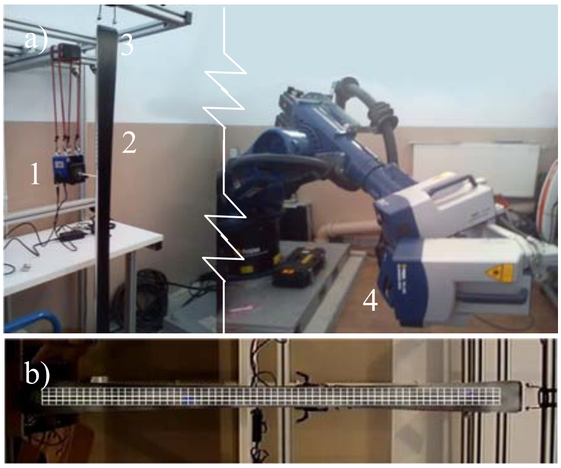


Fig. 2. a) The experimental setup: 1) the electromagnetic shaker, 2) the ski, 3) suspension, 4) laser vibrometer; b) the layout of the measurement points marked on the ski surface

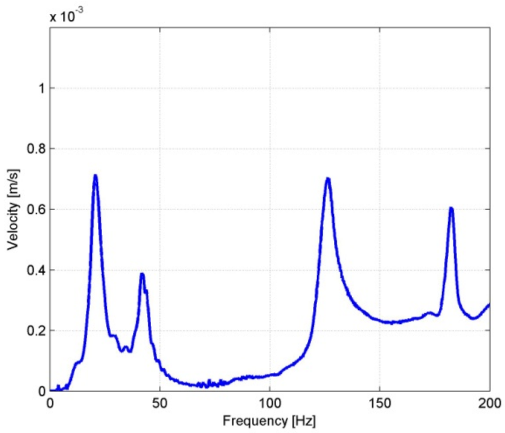


Fig. 3. The measured FRF for a point close to the ski tip

3. NUMERICAL MODEL

The commercial Finite Element (FE) software Abaqus was used in the current investigations. The geometry and internal structure of the ski has been reverse engineered based on a number of cross sections of one ski that was cut into pieces. Material properties have been adapted from the literature and assumed homogenous. FE grid convergence test has been performed in the model verification stage. As a result the working mesh consisting of 31114 degrees of freedom was used. The comparison between experimentally obtained and numerically predicted natural frequencies is shown in Tab 1. As can be seen, some modes predicted in a numerical model were not identified experimentally. That is due to the fact that the modes in question were the torsional modes and the assumed experimental setup with centrally attached shaker and vertical position of the ski prohibited the correct identification of those modes. This was, however, not an issue as the modes of interest that influence the traction properties of a ski are the bending modes. Numerical results presented in Tab.1 were obtained after some manual fine tuning of the material densities that was required to improve the agreement in natural frequency values. To identify the corresponding modes, between the numerical and experimental datasets, the Modal Assurance Criterion was used (Fig 4). As can be seen, the modes of interest (first three bending modes) are easily distinguishable.

Table 1. Comparison between experimentally obtained and numerical natural frequencies

Mode #	Simulation	Experiment
1	20.85	20.78
2	41.61	41.72
3	78.26	N/A
4	106.02	N/A
5	128.36	126.4
6	138.73	N/A
7	189.59	182.30
8	250.60	249.20
9	260.07	255.20

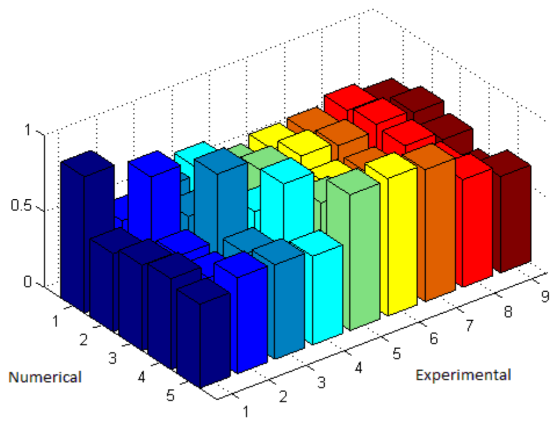


Fig. 4. MAC values for modes of interest between experiment and numerical model

4. NUMERICAL SHUNT DAMPING PERFORMANCE ASSESSMENT

Method proposed by Hagood and von Flotow [6] based on the concept of mechanical impedance was chosen as a calculation scheme. The reason for this choice is that the calculations based on modal approach for larger structures results in shorter model-to-results time especially when frequent circuit parameters changes are required.

The impedance method states that response of the system with piezo and RL shunt circuit may be described as:

$$\frac{x}{x_{st}} = \frac{\delta^2 + \gamma^2 + \delta^2{}_{ry}}{(1+\gamma^2)(\delta^2 + \gamma^2 + \delta^2{}_{ry}) + K_{ij}^2(\gamma^2 + \delta^2{}_{ry})} \tag{1}$$

where:

$$\delta = \frac{\omega_\varepsilon}{\omega_n},\ \omega_\varepsilon = \frac{1}{\sqrt{LC^s}},\ \gamma = \frac{s}{\omega_n},\ r = RC^s\omega_n,$$

$$K_{ij}^2 = \left(\frac{K_{jj}^E}{K + K_{jj}^E}\right)\left(\frac{k_{ij}^2}{1-k_{ij}^2}\right) = \overline{K}\left(\frac{k_{ij}^2}{1-k_{ij}^2}\right), \tag{2}$$

ω_n - vector of system natural frequencies,
s - Laplace variable,
L, C^s - impedance and inherent capacitance (at constant strain) of the patch respectively,
k_{ij} - electromechanical coupling coefficient,
K_{ij}^2 - generalized electromechanical coupling coefficient,
$\overline{K}$ - ratio of piezoelectric short circuit modal stiffness to the total system modal stiffness, or strain energy fraction.

After simple algebra transformations and accounting for any shunting circuit topology represented by arbitrary impedance Z the relation may be rewritten:

$$\frac{x}{F_{exc}} = \frac{1}{Is^2 + \varpi_n^2 + 2j\zeta\omega_m s + K_{ij}^2\varpi_n^2(\frac{ZC^s s}{ZC^s s+1})} \tag{3}$$

which may be further rewritten for the modal model:

$$\frac{x}{F_{exc}} = \frac{1}{M_{\mathrm{mod}}s^2 + K_{\mathrm{mod}} + C_{\mathrm{mod}}s + K_{ij}^2\varpi_m^2(\frac{ZC^s s}{ZC^s s+1})} \tag{4}$$

where:
ζ - modal damping
Z - electric impedance of shunting circuit

Values of generalized coupling coefficient may be obtained by calculating elastic strain energy fraction that is stored in the piezoelectric element for subsequent modes. According to Preumont [7] the values of generalized coupling coefficients give a descent approximation of the maximum damping added to the structural modes i. e.

$$\xi = \sqrt{\frac{K_{ij}^2}{2}} \tag{5}$$

$$K_{ij}^2 = \frac{\left(\omega_n^D\right)^2 - \left(\omega_n^E\right)^2}{\left(\omega_n^E\right)^2} \approx \frac{k^2 v_i}{1-k^2} \tag{6}$$

The theoretical values of the shunt circuit elements can be calculated for the RL single mode resonant circuit as follows [6]:

$$r = R_i C_{pi}^S \omega_n^E\ ,\ \omega_n^E \frac{1}{2\pi\sqrt{LC_{pi}^S}} \tag{7}$$

Where:
R_i – resistance of the PZT element,
$C^S{}_{pi}$ – capacity of the PZT element under constant stress,
$\omega^E{}_n$ – natural frequency with PZT element short circuited,
L – inductance value.

Despite the given theoretical values of RL circuit it was easy to notice that circuit impedances might have been manually adapted for better performance. As a consequence the theoretical values has been used as the starting point for optimization problem. The well known Nelder-Mead approach has been exploited defining four target functions to be minimized:

$$TF(RST) = \sum RST^2 \tag{8}$$

$$TF(RST) = \sum (RST - REF) \tag{9}$$

$$DIF = RST - REF \tag{10}$$

$$TF(RST) = \sum (DIF)(1 - sign(DIF)/2 \times NC + (1 + sign(DIF))/2 \times PC$$

$$TF(RST) = \max(RST) \tag{11}$$

Where:

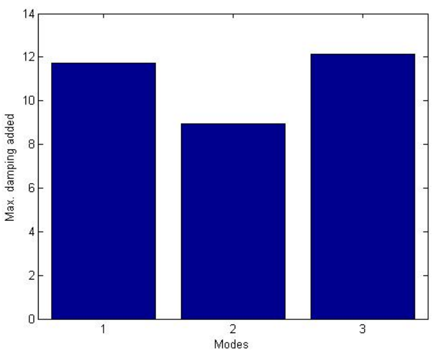
TF(RST) – target function to be minimized,
RST – frequency spectrum amplitude at given frequency point,

REF – reference frequency spectrum amplitude at given frequency point,
NC – coefficient controlling the height of the sidebands,
PC – coefficient controlling the depth of the central frequency.

Table 2. Maximum achieved damping in first three vibration modes. The four target functions TF (8-11) have been used for fine tuning of the shunt circuit parameters

TF / Mode	1st	2nd	3rd	4th
1st	4.84 dB	4.78 dB	3.58 dB	5.06 dB
2nd	2.78 dB	2.75 dB	1.96 dB	3.28 dB
3rd	4.14 dB	4.09 dB	3.02 dB	4.73 dB

At any frequency of interest the most efficient target function was the one which minimizes the maximum spectrum amplitude at a chosen frequency range (Tab 2.). At the same time the estimate of maximum added damping that base on the strain energy fraction stored in the PZT was compared with the actual decrease in response amplitude for particular modes. The comparison has been shown on Fig. 5 below.



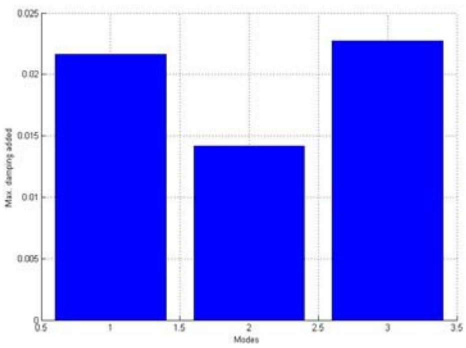


Fig. 5. Comparison between theoretical maximum added damping in terms of modal damping (Bottom) and actual decrease in response amplitude in dB (Top) for the third shape of the piezo patch

5. RESULTS ANALAYSIS

Strain energy fraction of modes stored in PZT patches, the PZT location as well as their area and shape are the most important factors when searching for the most efficient shunt damping solution. To be able to take all those factors into account the three different patches locations and two different shape (but of the same area) of patches were examined (Fig. 6).

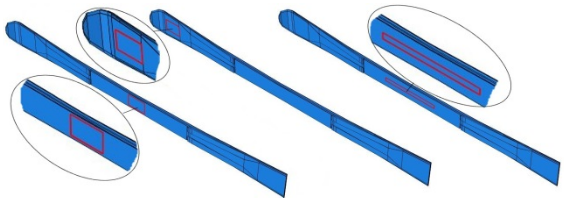


Fig. 6. Placement and shape of PZT patches

The achieved damping for each PZT location and first three normal modes has been shown in Tab. 3. The highest reduction in vibration amplitude for each mode has been obtained for narrow patch placed under the binding.

Table 3. Achieved damping for each PZT location for first three normal modes

Shape / Mode	1st	2nd	3rd
1st	5.06 dB	3.28 dB	4.73 dB
2nd	1.45 dB	1.96 dB	4.08 dB
3rd	11.74 dB	8.96 dB	12.12 dB

The shunt circuits parameters optimization procedure gave the response shape with balanced width and aligned height of the two resultant peaks (Fig. 7). This could not be achieved when using only theoretical values of the circuit parameters.

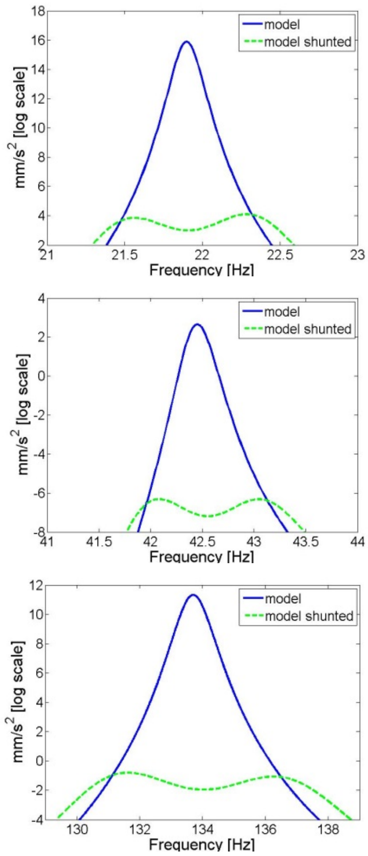


Fig. 7. Experimentally obtained system response (blue) and simulated shunted response (green) for the first three modes for the patch under the binding

The placement of the patches was arbitrary chosen based on the known mode shapes. The skis producers often induce the damping by some viscoelastic insertions which influence the inherent damping level of the skis. The more inherent damping the structures has the less effect is achievable by piezoelectric shunt damping system. The results of hypothetical study on the shunt damping effect on the ski having different level of damping has been shown in Tab. 4. It is clearly visible that passive piezoelectric damping treatment gives any relevant improvement only when paired to the lightly damped structures. This eliminates the possible use of hybrid methods and drives the attention toward active methods of piezoelectric damping.

Table. 4. Results of hypothetical values of damping with the shunt system for different values of inherent ski damping

Modal damping	0.005	0.1	0.01	0.05
Possibe treatment dB	5.06	0.17	2.67	0.43

6. RESULTS AND DISCUSSIONS

The presented study shows how the simple passive RL circuit can influence the dynamics of the ski. It was shown how the location and shape of the PZT patch may influence the maximum achievable damping of the ski modes. It was shown that fraction of mode's strain energy can be a good approximation of the shunt damping system performance. What is especially important when deciding on the location and shape of the PZT patches. For the arbitrary chosen ski it was possible to lower frequency spectrum amplitude of 11.45 dB, 8.96 dB and 12.12 dB for the first three modes respectively. The piezoelectric shunt damping system may be an asset for ski traction control, however, one should remember that any shift in system frequency will result in out of tune operation of resonant circuit. Therefore, only semi-passive embodiment of the PZT shunt can be of practical importance as they allow adaptive behavior. The expected yield of semi-passive (synthetic inductance based) system equals that of its passive counterpart. On the other hand the semi-passive systems are characterized by much lower energy consumption than active methods.

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