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MODELLING OF THE TEETH SEPARATION STATES IN GEARS CONSIDERING DIESEL ENGINE FORCING FEATURES

Jarosław DYK
Warsaw University of Technology, Institute of Machine Design Fundamentals, Warsaw, Poland
e-mail: jaroslaw.dyk@gmail.com

Summary

In the paper the possibility of the direct application of forcing computed function based on the elementary model of the diesel engine regarding issues of the rattle vibrations in modelling of gears was introduced. In this way the simulated indicator diagrams were created. After that, build diagrams were used to calculate tangential forces and torques operating in the drive system. Considering only the torsional vibrations, these calculated values of torques can be introduced as the forcing that are acting on gears inside of the gearbox. The presented in the article different models enable us to diagnose and analyse the neutral gear rattle. The forcing of the diesel engine introduced at the low range of rotation speeds is one of the most important parameters which may cause, within the clearance, the creation of the phenomenon of a lack of contact of the regular meshing of teeth, and, as a consequence vibrations and the difficult acoustic effects connected with rattle.

Keywords: gears, forcing, rattling, modelling

MODELOWANIE STANÓW UTRATY KONTAKTU ZAZĘBIEŃ KÓŁ ZĘBATYCH Z UWZGLĘDNIENIEM WŁAŚCIWOŚCI WYMUSZENIA SILNIKIEM SPALINOWYM O ZAPŁONIE SAMOCZYNNYM

Streszczenie

W pracy, dla zagadnień dotyczących drgań rattle w modelowaniu przekładni, przedstawiono możliwość zastosowania bezpośredniego wyliczania funkcji wymuszającej silnika na podstawie elementarnego modelu uwzględniającego zmiany ciśnienia czterosuwowego silnika o zapłonie samoczynnym. Stworzono w ten sposób symulowane wykresy indykatorowe, które wykorzystano do obliczeń przebiegów sił stycznych i momentów działających w układzie napędowym. Zakładając jedynie drgania skrętne obliczany moment wprowadzić można jako wejściowe wymuszenie przekładni znajdujących się wewnątrz skrzyni biegów. Przedstawiono różne modele opisujące klekotanie zębów w biegu luzem, które umożliwiają diagnozę powstania tego zjawiska. Wymuszenie od silnika działające w niskim zakresie prędkości obrotowych jest jednym z ważniejszych parametrów, który spowodować może, w obrębie luzu międzyzębnego, powstanie zjawiska utraty prawidłowego styku zębów, a w konsekwencji drgań i uciążliwych efektów akustycznych związanych z klekotaniem.

Słowa kluczowe: przekładnie zębate, wymuszenie, klekotanie, modelowanie

1. INTRODUCTION

In the toothed wheels the backlash in mesh, as a one of the nonlinearities in the system, cause sometimes an effect on a loss of the contact between the meshing teeth and it may generate the vibration (so-called *rattling*, *rattle*). This effect can be attributed to the teeth engagements on the theoretically correct line of action, to the separation of meshing teeth inside the backlash space or to the mesh on the instantaneous second line of action (which corresponds to the reversible motion of the gear). There are intermediate states of the positions of the teeth when they move separately inside the backlash zone too (e.g. only the departure from the regular motion exists -so called one–side impacts of the teeth).

Such behaviour of the teeth increases levels of vibration and noise. It can be one of the crucial problems in unloaded and lightly loaded gears e.g. in vehicles for gearboxes and for machine tools etc. It means, that it is one of the NVH problems (noise, vibration and harshness issues). This way the rattle phenomenon ought to be at first diagnosed to forecast the possible levels of the vibration and noise.

These vibrations were a subject of many research works (e.g. [1-3,5-9,11-15]). The parameters that have an impact on the generation of the rattle vibration, besides backlash, are:
- variable vibration forcing,
- zones of rotational velocity.

There are also some cases for which the vibration problems due to rattle vibrations e.g. gears with significant base pitch deviations, gears operated at resonant zones by the constant external loads [6] and gearboxes in vehicles for idle running (*neutral gear rattle*) occur.

The focus of this report is to show the physical structure and the examples of results of performed calculations for two different models that can be used for searching the rattle conditions for the diagnostic and rattle prediction purposes.

2. DESCRIPTION OF THE FORCING FUNCTION

Usually, for the steady state (if we use the experimental data for the fixed position of the throttle) we can obtain the plot of the expanded indicator diagram. In the computational research studies sometimes knowing the driveline we assume the approximate information about engine and its parameters that can be changed arbitrarily for simulation purposes. In such cases, based on the elementary theoretical model of the diesel engine, the indicator diagram may be obtained. An example of the simplified schema of the indicator diagram for the diesel engine is shown in Fig.1.(cf. [10]), where, p -pressure, S=2R, R-radius of the crank, p_a-atmospheric pressure. In this way the simulated indicator diagrams may be created with a sufficient accuracy, which depends on the way of the description of the lines and curves in given simulation (Fig. 2). In this figure for non-dimensional values of s the non-dimensional pressure p/p_4 is shown. There is an easy access to some of the pressure parameters and characteristic angles of engine cycles etc. in both baseline and scientific literature.

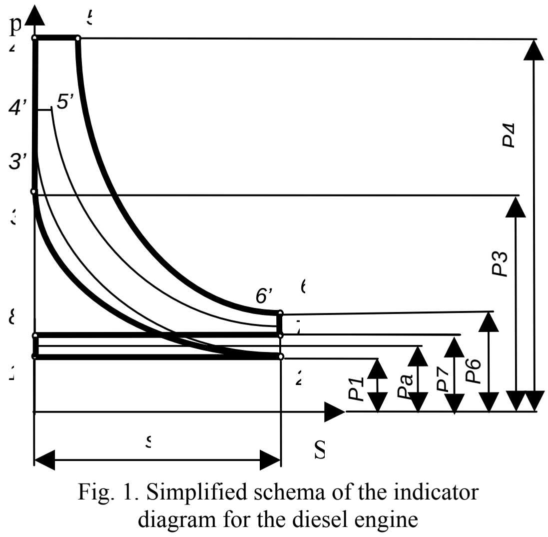

Fig. 1. Simplified schema of the indicator diagram for the diesel engine

In case of a diesel engine cycle for the compression (pressure in points 2-3) and expansion (3-4-5-6), for pressure points (5-6) applicable curves values rely on the position of a piston and the polytropic (isentropic) exponent (index) must be calculated. The statistic data for the values ranges of this index is available in the literature. Except the pressure values in the characteristic points some parameters such as a compression ratio, an arm of the crankshaft, the geometric ratio between the arm of crankshaft and length of the connecting rod etc. are obligatory.

Continuing, it is possible to calculate the input torque function (e.g. Fig. 3) based on the tangential loading which acts on the single journal of a crankshaft (the crank pin efforts). Considering the number of cylinders the suitable superposition of this function for four cylinders diminishes the period of this function from 720 deg to 180 deg (Fig. 4).

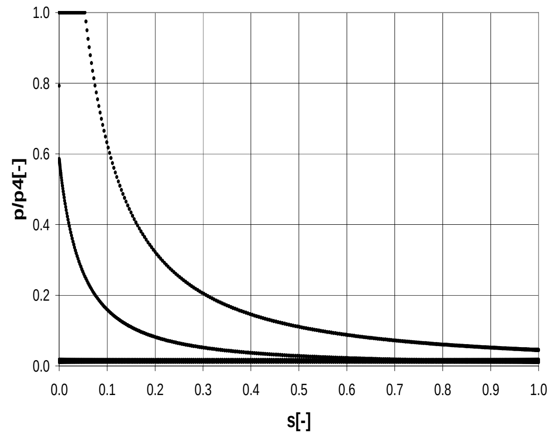

Fig. 2. Simulated values of the non-dimensional indicator diagram for one cylinder of the diesel engine (cf. [10])

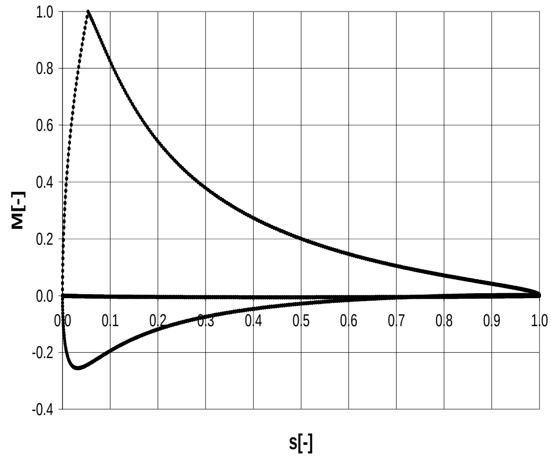

Fig. 3. Example of the simulated non-dimensional indicator diagram for one cylinder of the diesel engine

We assume the forcing of the driveline just on the output of the crankshaft as an area of article interest. Usually in this place there is a flyway and a single- or dual-stage clutch. For purposes of the further simulation of the rattling processes we neglect the interaction all the crankshaft vibrations on the rest of a driveline (similarly to the authors of [3, 5, 14]).

There is also another issue, which influences on the description of the forced vibration of the driveline. Having the variable torque (that can be calculated on the basis of the indicator diagram, geometrical dimensions of the crank system and mass distribution for specific rotation speeds) we can calculate the varying inertial forces and these forces may be added. However, these forces are usually partly balanced (it depends on the kind of the engine and its construction features) and these unbalanced loads may produce additional varying torque. In the calculation both cases may be adapted but at the low range of the rotation speeds (e.g. for the rattle issues) the inertial forces have not significant values. However, it is possible to make an assumption that the engine is unbalanced, so in this way, the worst case of the forcing is estimated.

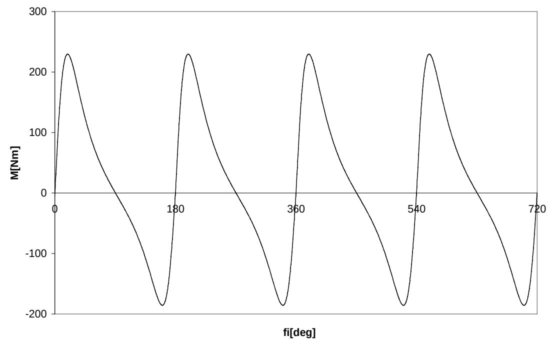

Fig. 4. Example diagram for the simulated torque four cylinder diesel engine

As it was mentioned above, during the simulation, the possibility of the straightforward application of forcing computed function based on the model of the diesel engine is introduced. However, this function can be computed separately. In such case, the obtained values are transformed by use of the harmonic analysis and introduced as Fourier series. Fig. 5 shows a comparison between the simulated engine torque and anew values of the same torque treated as data calculated by use of Fourier series. In this figure the thick and thin lines represent simulated engine torque and its approximation. For this calculation sixteen terms of the series were enough. In reality, the values of the engine torque are not so regular as in a case of the standard periodic function. The real working cycle of an engine is different from theoretic one for thermodynamic reasons mainly and this phenomenon reflects a typical indicator test which is carried out experimentally. In the practical realization of the task using computer simulation the assumption about the ranges of pressure for the characteristic points on the simulated indicator diagram must be recognized.

In fig. 1 these points and ranges of the fluctuation of the parameters are shown as additional cycle borders for the compression (pressure in points 2-3') and expansion (3'-4'-5'-6') for pressure in points (5'-6'), therefore the new (isentropic) exponents (index) should be employed. The random character of this process has been accepted. Practically, during the computer simulation the generators of pseudo- random numbers may be used. The obtained in this way values should be tested and may be changed. The values of the exponents are changed within real range of the permissible limits. Essential difficulty is encountered in establishing the parameters of the point 5' and 6'. For these points the exponent must be calculated by use of logarithmic equations and verified with the value of the pressure parameter in point 7. The Fig. 6 presents an example of the simulation of torque for random conditions.

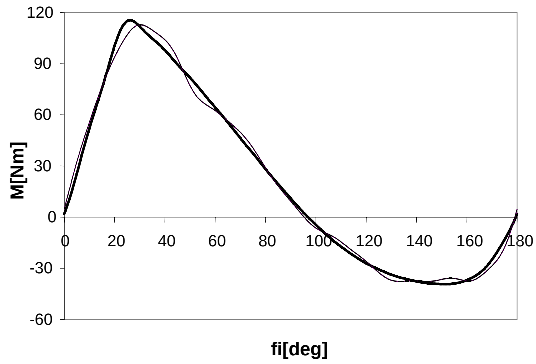

Fig. 5. A comparison between the simulated torque (thick line) and the values of the same torque approximated by use of Fourier series (thin line)

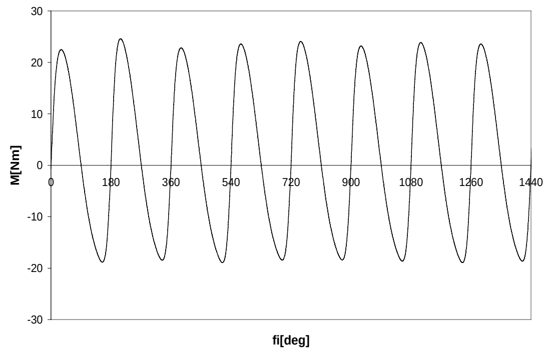

Fig. 6. Example of the simulation of torque for random conditions

3. GEAR MODELS – NUMERICAL EXAMPLES

Two models for the aspect of the issues considering gear rattle and description of the engine forcing are presented. These models are useful in conducting the research work into the rattle phenomena in gears of gearboxes. The first one is show in Fig. 8. The equations describing the model are generally compatible with the form from [14, 3, 5] and can be written as

$$I_1 \frac{d^2\varphi_1}{dt^2} + T_{cv}\left(\frac{d\varphi_1}{dt}, \frac{d\varphi_2}{dt}, \varphi_1, \varphi_2, c_{c1}, c_{c2}\right) + $$
$$+ T_c(\varphi_1, \varphi_2, k_{c1}, k_{c2}, H_{c1}, H_{c2}) = T_e(t)$$
$$I_2 \frac{d^2\varphi_2}{dt^2} - T_{cv}\left(\frac{d\varphi_1}{dt}, \frac{d\varphi_2}{dt}, \varphi_1, \varphi_2, c_{c1}, c_{c2}\right) + $$
$$+ c_s\left(\frac{d\varphi_2}{dt} - \frac{d\varphi_3}{dt}\right) - T_c(\varphi_1, \varphi_2, k_{c1}, k_{c2}, H_{c1,} H_{c2}) + $$
$$+ k_s(\varphi_2 - \varphi_3) = \mathbf{0}$$

$$I_3\frac{d^2\varphi_3}{dt^2}-c_s\left(\frac{d\varphi_2}{dt}-\frac{d\varphi_3}{dt}\right)+$$
$$+c_g r_3\left(t,r_3\frac{d\varphi_3}{dt}-r_4\frac{d\varphi_4}{dt}\right)+c_h\left(\frac{d\varphi_3}{dt},\Omega_e\right)+$$
$$-k_s(\varphi_2-\varphi_3)+k_g(t)r_3F\left(t,r_3\varphi_3-r_4\varphi_4,x_{er},x_\eta\right)=\mathbf{0}$$
$$I_4\frac{d^2\varphi_4}{dt^2}-c_g r_4\left(t,r_3\frac{d\varphi_3}{dt}-r_4\frac{d\varphi_4}{dt}\right)+$$
$$+c_h\left(\frac{d\varphi_4}{dt},\Omega_e/u\right)-k_g(t)r_4F\left(t,r_3\varphi_3-r_4\varphi_4,x_{er},x_\eta\right)=\mathbf{0} \qquad (1)$$

In the equations (1) the phenomena of non-Newtonian behaviour of the oil film on the surfaces of teeth are neglected (cf. [14]). In the system the description of the proportional damping similar to the spring forces in the mesh and general dependence of the level of damping c_g on the zone of the mesh (normal engagements of teeth and momentary for reversible motion, dead zone of backlash) is adapted. Dissipative forces are obviously present in another parts of the model e.g. in the mathematical model in Singh's and his co-workers' paper [14] where there are introduced the drag torques, which are tied with each wheel in the gear stage and is described the form of sums of components which are dependent of the rotational speed Ω_i and suitable vibration velocity (factor c_h). It is worth mentioning that proper distribution of dissipation among the all parts of the system and correct evaluation of the dissipative forces are crucial for the results of the simulation. A temporary lack of contact of engaged teeth results in impacts (so-called one-side and two-side impacts). Excluding deviations (errors) relevant to corresponding flanks of the gear teeth, the mesh function for stiffness in translational co-ordinates for relative displacements between gears (for a given stage of the gear $x=r_i\varphi_i-r_{i+1}\varphi_{i+1}$) can be defined and written in the form:

$$F(x,x_\eta)=\begin{cases} x, & x>0 \\ 0, & -x_\eta\le\ x\le 0 \\ x+x_\eta, & x<-x_\eta \end{cases} \qquad (2)$$

The description of the variable gear meshing stiffness for spur or helical gears [2] and of deviations relevant to corresponding flanks of gear teeth is also introduced in the equations describing this model. Later we assume that there is a dual stage clutch (dry friction clutch[14, 3]) with non-linear characteristics with hysteresis (H_{c1}, H_{c2}) and without (or with) hysteresis of the second stage (the torques T_{cv} i T_c in symbolic description) in the system. Moreover, in the equations (1) we apply the following notation: J_i - moment of inertia (i = 1, 2, 3); I_i- moments of inertia of the flywheel, the clutch hub, the input gear (equivalent value) and the output gear respectively, (cf. Fig. 7, i = 1, 2, 3, 4); $r_3 = r_{b3}$, $r_4 = r_{b4}$ - base radii; c_{c1}, c_{c2}, c_s, c_g - viscous damping in clutch, in shaft, and equivalent value in the mesh zone; k_{c1},k_{c2}, k_s - stiffness by analogy with the foregoing; $k_g(t)$- global meshing stiffness- the value calculated for each zone of the mesh; F- symbolic description of the mesh function which depends on time, on the mesh zone, on static deflections, on current difference between values of single pitch deviations for the pinion and wheel ψ_{er} as well as on backlash ψ_η (in Eqs (1) values x_{er} and x_η respectively [4]). In calculation value of I_3 should include all the gear wheels that are in reality behind I_3 except for the value I_4, remaining unchanged. In this way we obtain the equivalent system in which the input gear has the moment of inertia considerably bigger than the one that describes the output wheel.

Input torque excitation has the form:

$$T_e(t)=T_{e0}+\sum_{l=1}^{n}T_l\sin[(N_e/2)l\Omega_e t+\varphi_{al}] \qquad (3)$$

where: N_e -number of cylinders; l=1,2,...; Ω_e- engine rotational speed; φ_{al} -phase. By simulation of the idle run there are taken only the first two engine harmonics and the mean engine excitation torque that may be assumed as equal to zero. In the computer simulations, usually the most important terms are employed, however, in the literature in such cases it is recommend to significantly greater number of harmonics.

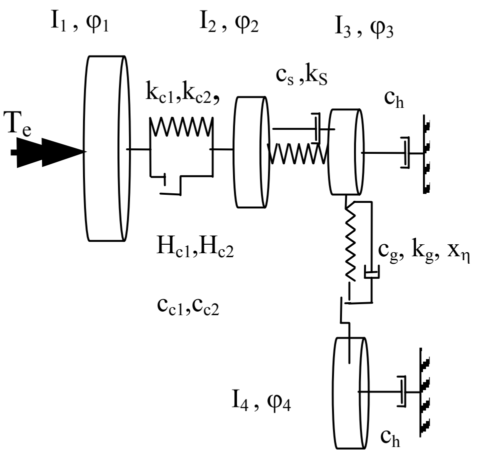

Fig. 7. Equivalent model of a driveline useful for analysing of the gear rattle

In research works, as we mention above, it is possible, that one can apply the method of direct computing of the input torque excitation by simulation instead of using the series form (and written form similar to formula (3)). Another solution which may be chosen depends on the means of the harmonic analysis. In this case, it is also possible to rely on the experimental data of the real structure or on the results of forcing for the simulated engine. As an example of the results Figure 8 presents the picture of a phase plain for the vibration within the backlash gap. A similar case is shown in Fig. 9. Both figures illustrate the state of the system in which gear rattle take place. Another examples and detailed information about majority of the parameters used in the computation are in [4, 5].

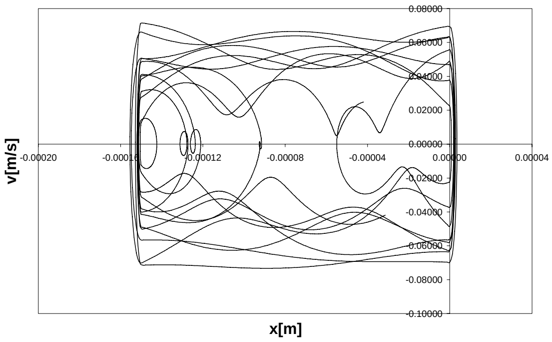

Fig. 8. Phase plain –gear rattle

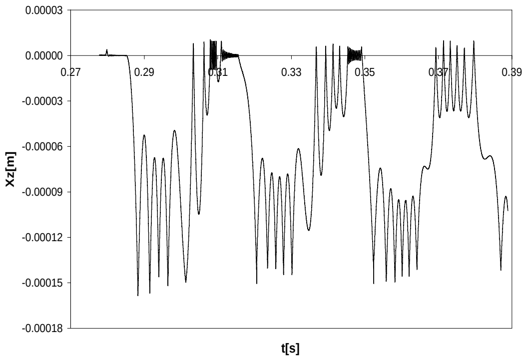

Fig. 9. Time history of vibrations–gear rattle

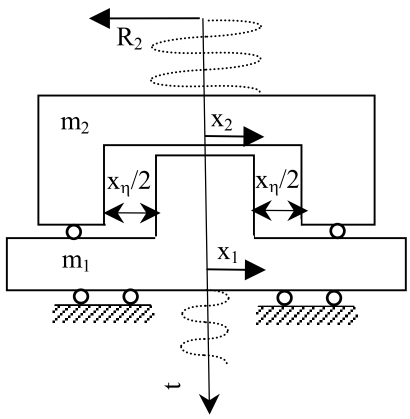

Fig. 10. Equivalent model for the analysis of the free toothed wheel impact processes the according to [16, 8]

The second model has a different structure and is written in the linear coordinates. In this model (Fig. 10) vibrations of the free element that has mass m_2 are generated by the impact process with the forcing element having mass m_1. Particularly, this element moves precisely according to the dependence describing by harmonic function

$$x_1(t) = \hat{x}_1 \sin(\omega_{\text{force}} t) \tag{4}$$

where, ω_{force} - engine forcing frequency, $\hat{x}_1$ - amplitude. Therefore, in the description of the forcing, components which are the results of irregularity of run for the engine are taken into account. If a condition for the backlash x_η is fulfilled

$$|x_2 - x_1| < \frac{x_\eta}{2} \tag{5}$$

then the equation describing the phase of flight has the form

$$m_2 \ddot{x}_2 = -R_2 \tag{6}$$

where, R_2 - external friction force. Reaching the border of backlash means that the impact takes place and the velocities of both elements are changed. This is illustrated by the formula (ε - coefficient of restitution)

$$\dot{x}_{2af} = \dot{x}_{1be}(1+\varepsilon) - \dot{x}_{2be}\varepsilon \tag{7}$$

where, $\dot{x}_{2af}$ -after the impact, $\dot{x}_{1be}$ - at the moment of the impact, $\dot{x}_{2be}$ - after the impact.

After the impact the free element either comes off from the forcing element, when

$$\ddot{x}_1 < -\frac{R_2}{m_2} \tag{8}$$

or the free element achieves a common velocity with the second element and in this way sticks one to another, when the mentioned below condition is fulfilled

$$\ddot{x}_1 \geq -\frac{R_2}{m_2} \tag{9}$$

The solution of this issue is possible after double integration of the equation (6) and fulfilling the initial conditions.

In case of forcing by harmonic function ([16], cf. [8]) the non-dimensional parameter of the torque C_M and dimensionless backlash factor C_s were introduced. Detailed procedure of using results of the mathematical solution of this model and the experimental data (the special stand) was introduced [16] for effective evaluation and prediction of the levels of noise connected with gear rattle.

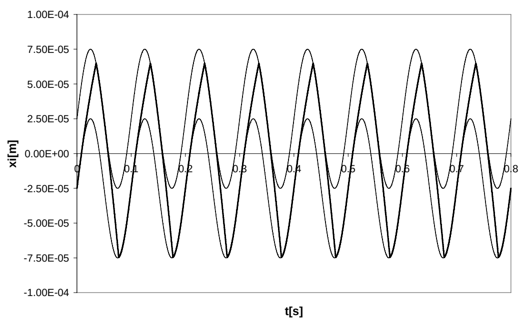

Fig. 11. Time history of the free wheel displacements with the restitution coefficient equal to zero

An example of the results of computation for this model by the coefficient of restitution ε =0 is shown in the Fig. 11. The computations were conducted for the given values of the parameters C_M=1, C_s=0,5 and the engine forcing frequency ω_{force}=20π rad/s. The evaluated mass m_1 included also the equivalent mass of the flywheel. The number of teeth for the pinion was equal to z_1= 20 and for the wheel z_2 =40. The module pitch m_n=2 mm. This figure illustrates the case, when after each impact teeth stick one to another.

It is worth pointing out that the issue of the vibration analysis of this model including the suitable selection of the parameters, can be reduced to the evaluation of the results of alternative equivalent system. This systems can be written in the form

$$\begin{aligned} &m_{1eq}\ddot{x}_1 + c_{1eq}\dot{x}_1 + k_{1eq}x_1 = T_e(t)/r_1 \\ &m_2\ddot{x}_2 + \bar{c}_h(\dot{x}_2, V) = 0 \end{aligned} \tag{10}$$

where, m_{1eq}-equivalent mass included the flywheel, clutch and all the gear wheels that are in reality behind m_1, c_{1eq} and k_{1eq}- equivalent viscous damping and stiffness, respectively, $\bar{c}_h$ -factor (like in Eqs (1)) dependent from the velocities of vibration and pitch-line V, T_e- external torque, r_1-base radius of the pinion. These formulae represent seemingly two independent equations with suitably chosen equivalent mass, damping and stiffness parameters in the first of Eqs (10) but the values of difference of x_1 and x_2 (like in eq. (5)) determine the solution of the second equation.

Similarly, for Eqs (1) the simplified form of the equivalent system can be created (Eqs (11)). In the presented form of these equations the parameters of the mesh stiffness varying with time $k_g(t)$ and recalculated damping $\bar{c}_h$ (as in Eqs (10)), as well as F that means the symbolic description of the mesh function which depends on time, on the mesh zone and on backlash x_η, are included.

$$\begin{aligned} &m_{1eq}\ddot{x}_1 - k_g(t)F(t, x_2 - x_1, x_\eta) + \bar{c}_h(\dot{x}_1, V) = T_e(t)/r_1 \\ &m_2\ddot{x}_2 + k_g(t)F(t, x_2 - x_1, x_\eta) + \bar{c}_h(\dot{x}_2, V) = 0 \end{aligned} \tag{11}$$

This approach needs detailed verification and comparison of the obtained simulation results with the results of the experiments.

4. CONCLUSIONS

One of the most important issues considering diagnosing and the evaluation of the vibroacoustic effects of the gear rattle on the basis of the analysis of the dynamic models, is the proper choosing of the significant physical and technical parameters. In the paper the role of the forcing from diesel engine in the systems with the backlash is highlighted. The models presented in the article allow us to diagnose and analyse the neutral gear rattle.

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Jarosław DYK, Phd, DSc. His scientific interests include dynamics of machine and dynamics of gears as well as machine design and numerical simulation.