



IDENTIFICATION OF MODAL PARAMETRS OF TRANSMISSION TOWERS

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Abstract

The paper presents theoretical basis for identification of nonlinear systems using the method based on Hilbert transform. Discussed method was verified by analyzing the synthesized signals of known parameters. Developed and verified mathematical tools were used for analysis of dynamic responses of the real object: a laboratory (physical) model of the transmission tower. For the considered laboratory model it was possible to simulate damages of individual truss elements and observe changes in system dynamic characteristics. Results of the carried out research proved that the first natural frequency and damping factor change in the observable extent as a result of damage presence.

Key words: transmission tower, modal parameters, Hilbert transform

IDENTYFIKACJA PARAMETRÓW MODALNYCH KONSTRUKCJI SŁUPA ELEKTROENERGETYCZNEGO

Streszczenie

W pracy przedstawione zostały teoretyczne podstawy identyfikacji układów nieliniowych z wykorzystaniem transformaty Hilberta. Omówioną metodę zweryfikowano przeprowadzając analizę syntezy sygnałów o znanych parametrach. Opracowany aparat matematyczny zastosowano do analizy odpowiedzi dynamicznych obiektu rzeczywistego: modelu laboratoryjnego (fizycznego) przemysłowej konstrukcji wsporczej znajdującej się w ciągłej eksploatacji. Rozpatrywany model laboratoryjny umożliwił symulowanie uszkodzeń poszczególnych elementów kratownicy i badanie ich wpływu na zmianę parametrów dynamicznych. W pracy wykazano, że w wyniku uszkodzenia układu wartości pierwszych częstotliwości drgań własnych oraz tłumienia zmieniają się w stopniu obserwowalnym.

Słowa kluczowe: słup elektroenergetyczny, parametry modalne, transformata Hilberta

1. INTRODUCTION

Methods of damage detection and identification are of key importance for physical systems condition assessment [1]. They make it possible to develop monitoring systems that find applications in both emerging and worn objects, which are very often on the border of operational lifetime. Faults detection at the early stages of their propagation makes it possible to detect impending emergency states and, through the adequate supervision of the object, minimize downtime periods [2, 3]. Nowadays, variety of different identification methods based on different physical phenomena is available. In the published papers [4], the authors proved the suitability of a particular method for the selected type of objects or a group of objects of similar structure. The selection of method strongly depends on the type of damage that may arise in the considered system, system structure and complexity

as well as material properties. Very often developed methods are 'tailor made' for given construction [5], making it possible to highlight certain features, which allows for more efficient evaluation of the system technical condition. In the last few years a lot of attention was paid to development of fault detection methods based on measurements of acoustical quantities, vibrations and also related to the propagation of waves in the considered system [6, 7]. It could be stated that these techniques are most efficient in condition assessment and monitoring, which is reflected in the range of their practical applications.

Vibroacoustical modulations are widely used in monitoring of multiple nonlinear symptoms based on the vibro-acoustical responses. Usually, a measure of modulation depth is directly related to the severity of damage. Different measures of defect size are defined based on changes in the amplitude, phase, carrier frequency and

modulations. The paper [8], dedicated to demodulation of envelope and instantaneous frequency, provides a time-domain analysis of modulated acoustical response of the tested structure. The work focuses on the analysis of response transient characteristics using Hilbert transform. In acoustical responses of the aluminium plate with the propagating crack, modulations of both amplitude and frequency appear. However, the depth of amplitude modulation is significantly higher than in case of frequency modulation.

The idea of the instantaneous frequency application as an indicator of damage presence has been studied by Bernal and Gunes [9]. The first step of the EMD (Empirical Mode Decomposition) method application consists in performing decomposition of the considered signal into harmonics that, in this method, are called IMF (Intrinsic Mode Functions). If after sudden appearance of a major damage the system still exhibits linear properties, then using this method it is possible to identify the damage size and time of its occurrence. Although the computations were carried out for the noiseless signal, the authors proved that the method is also efficient in real (exploitational) measurement conditions. In case of responses of hysteretic systems, the value of the instantaneous frequency as an indicator of moderate nonlinearity is not very sensitive. For significant nonlinearities the values of instantaneous frequency were strongly related to the non-linear characteristics of the system stiffness force. Analysis of instantaneous frequency was used to interpret the nonlinear responses of the rotor with propagating damage [10].

EMD (Empirical Mode Decomposition) method for evaluation of the technical condition was also used by Salvino et al. in [11]. They examined features of time-frequency and instantaneous phase characteristics for changes that may occur due to damage propagation. Based on the time-frequency analysis and simple mathematical models, the authors have demonstrated that the EMD method (in particular the instantaneous phase detection) can be used to identify the presence and location of the structural damage.

Application of the instantaneous damping factor to identification of damages in beams and frame structures has been proposed by Curadelli et al. [12]. They applied wavelet transform to analyze time histories obtained on the basis of the frame structure mathematical model and a real simply supported beam. Both numerical tests and experiments have proved that object damage results in significant changes in damping characteristics and relatively small changes in natural frequencies. Therefore the values of damping factor or other, more complex metrics formulated on the basis of damping factor, can be used for the purposes of damage detection.

2. IDENTIFICATION OF MODAL PARAMETERS WITH THE APPLICATION OF HILBERT TRANSFORM

2.1. Analytical Signal

Identification of modal parameters with the application of Hilbert transform is based on the signal representation in the form $X(t) = x(t) + j\tilde{x}(t)$, where $\tilde{x}(t)$ denotes HT (Hilbert Transform) projection based on Hilbert transform of function $x(t)$ (Fig. 1).

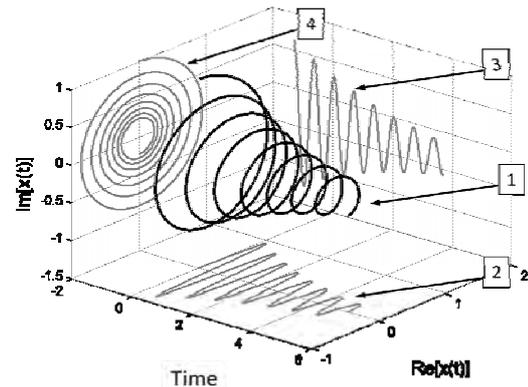


Fig. 1. Example complex signal (1), real part (2), imaginary part (3), phasor on the complex plane (4)

The method uses the signal envelope and polar form representation $X(t) = A(t)e^{j\psi(t)}$, where $A(t)$ denotes envelope and $\psi(t)$ - instantaneous phase. Both functions are real-valued: $x(t) = A(t)\cos\psi(t)$, $\tilde{x}(t) = A(t)\sin\psi(t)$, $A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}$, $\psi(t) = \arctg\left[\frac{\tilde{x}(t)}{x(t)}\right]$. System response is determined by

direct measurements of vibration time histories while HT projection $\tilde{x}(t)$ is obtained by the application of Hilbert transform. Signal envelope, phase and their derivatives can be calculated directly as functions of time or on the basis of relations determined for the complex signal:

$$\dot{X} = X \left(\frac{\dot{A}}{A} + i\dot{\psi} \right), \quad \ddot{X} = X \left(\frac{\ddot{A}}{A} - \dot{\psi}^2 + \frac{2i\dot{A}}{A}\dot{\psi} + i\ddot{\psi} \right), \quad \text{where}$$

$$\dot{\psi}(t) = \omega(t) = \frac{x(t)\tilde{x}'(t) - \dot{x}(t)\tilde{x}(t)}{A^2(t)} = \text{Im} \left[\frac{\dot{X}(t)}{X(t)} \right] \quad \text{is an}$$

instantaneous frequency of signal $x(t)$.

Let's consider the nonlinear equation of motion of conservative oscillating system with damping in the following form:

$$\ddot{x} + h_0(\dot{x})\dot{x} + \omega_0^2(x)x = 0 \quad (1)$$

It is known that nonlinear elastic force expressed as a function of time can be converted to the form: $\omega_0^2(x)x = \omega_0^2(t)x(t)$ with the new fast-

varying natural frequency $\omega_0^2(t)$ and the signal $x(t)$, for which the spectra coincide. Similarly, non-linear damping force can be converted into a time-dependent form: $h_0(\dot{x})\dot{x} = h_0(t)\dot{x}(t)$ that is the product of fast-varying instantaneous damping coefficient $h_0(t)$ and velocity of coinciding spectra. As a result, instantaneous natural frequency $\omega_0^2(t)$ and instantaneous damping factor $h_0(t)$ are obtained. Spectra of instantaneous natural frequency $\omega_0^2(t)$ and instantaneous damping factor $h_0(t)$ are broadband and satisfy the equation:

$$\ddot{x} + 2h_0(t)\dot{x} + \omega_0^2(t)x = 0 \quad (2)$$

Both instantaneous natural frequency and instantaneous damping factor can be separated into low and high-frequency components, in such a way that they spectra do not overlap. Applying Bedrosin identity:

$$H[n_{slow}(t)x_{fast}(t)] = n_{slow}(t)\tilde{x}_{fast}(x) \quad (3)$$

where n_{slow}, x_{fast} denote slow and fast-varying (low and high-frequency) functions of separated spectra, respectively. Hilbert transform of separated components:

$$\begin{aligned} H[(2\bar{h}_0(t) + 2h_1(t))\dot{x}(t)] &= 2\bar{h}_0\dot{\tilde{x}} + 2\tilde{h}_1\dot{x}(t) \\ H[(\bar{\omega}_0^2(t) + \omega_1^2(t))x(t)] &= \bar{\omega}_0^2\tilde{x} + 2\tilde{\omega}_1^2x(t) \end{aligned} \quad (4)$$

As a result, the Hilbert transform of motion equation (Eq. 2) takes the following form:

$$\tilde{\ddot{x}} + 2\bar{h}_0(t)\tilde{\dot{x}} + 2\tilde{h}_1(t)\dot{x} + \bar{\omega}_0^2(t)\tilde{x} + \tilde{\omega}_1^2(t)x = 0 \quad (5)$$

where: $\bar{h}_0, \bar{\omega}_0$ – slow varying damping factor and natural frequency, respectively.

Multiplying the (Eq. 5) by i and adding equations (Eq. 2) and (Eq. 5), differential motion equation defined for analytical signal is obtained [13]:

$$\dot{X} + 2h(t)\dot{X} + \omega_x^2(t)X = 0 \quad (6)$$

where $\omega_x(t)$ denotes instantaneous natural frequency, $h(t)$: instantaneous damping factor and $X = x(t) + i\tilde{x}(t)$. Both instantaneous parameters of the above equation are unknown.

However, the equations have a complex form consisting of two separate parts: real and imaginary. Therefore the number of equations is equal to the number of unknowns, so the system of equations has one solution for instantaneous modal parameters.

2.2. Modal characteristics of free vibrations

Let's consider free vibrations of nonlinear SDOF (Single Degree of Freedom) system. Second order system with nonlinear stiffness and nonlinear damping has a solution in the form of analytical

signal $X(t) = x(t) + i\tilde{x}(t)$, where $x(t)$ is a measured response of the considered vibrating system.

In the course of identification, using Hilbert transform in time domain, analytical signal representation $X(t) = x(t) + i\tilde{x}(t)$ is used, where $\tilde{x}(t)$ denotes Hilbert transform of signal $x(t)$. The method uses signal envelope and signal phase representation $X(t) = A(t)e^{i\psi(t)}$ where $A(t)$ is an envelope and $\psi(t)$ is an instantaneous phase, respectively. Both functions are real: $x(t) = A(t)\cos\psi(t)$, $\tilde{x}(t) = A(t)\sin\psi(t)$, $A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}$, $\psi(t) = \arctg\left[\frac{\tilde{x}(t)}{x(t)}\right]$. System

responses are determined on the basis of direct vibration measurements, while HT projection $\tilde{x}(t)$ is computed by the application of Hilbert transform. Derivatives \dot{X} and \ddot{X} of complex analytical signal X are known functions of amplitude $A(t)$ and frequency $\omega(t)$ that can be written as follows:

$$\dot{X} = \dot{A}(t)e^{i\psi(t)} + i\omega(t)A(t)e^{i\psi(t)} = X\left[\frac{\dot{A}}{A} + i\omega\right] \quad (7)$$

$$\ddot{X} = X\left[\frac{\ddot{A}}{A} - \omega^2 + 2i\frac{\dot{A}}{A}\omega + i\dot{\omega}\right] \quad (8)$$

Substituting derivatives into (Eq. 6) it can be written that:

$$X\left[\frac{\ddot{A}}{A} - \omega^2 + \omega_x^2 + 2h\frac{\dot{A}}{A} + i\left(2\frac{\dot{A}}{A}\omega + \dot{\omega} + 2h\omega\right)\right] = 0 \quad (9)$$

Separating real and imaginary parts and comparing them to zero, instantaneous modal parameters can be determined:

$$h(t) = -\frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega} \quad (10)$$

$$\omega_x^2(t) = \omega^2 - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega} \quad (11)$$

where A : signal envelope, ω_x estimated instantaneous frequency. Both obtained instantaneous modal parameters – natural frequency $\omega_x^2(t)$ and damping factor $h(t)$ – are the functions of the first and the second derivative of signal envelope and frequency. Instantaneous modal parameters can be directly estimated for each measured sample in the vibration signal $x(t)$. The above equations mean that the method for identifying system parameters based on the Hilbert transform uses displacements, velocities and accelerations as an input. Due to the method assumptions and the way of formulating motion equations, determined parameters are independent from the type of nonlinearity present in the analyzed system. In order to obtain backbone curve

it is necessary to combine instantaneous natural frequency and envelope. Similarly, while combining damping with envelope, damping curve can be obtained. Damping and backbone curves are used as a basic tool in the analysis of nonlinear vibrations.

It should be mentioned that natural frequency $\omega_x^2(t)$ differs from the instantaneous natural frequency, since it depends on variations in signal envelope and the instantaneous natural frequency itself. For small and slow varying nonlinearities, e.i. when the second order components can be neglected (under assumption that $\dot{A}^2 = \ddot{A} = \dot{\omega} = \dot{A}\dot{\omega} = 0$), on the basis of equation defining the modal parameters, it can be stated that natural frequency is close to the instantaneous natural frequency while instantaneous damping factor equals the ratio of signal envelope to its derivative.

When nonlinearities result in changes in signal envelope and instantaneous natural frequency, instantaneous natural frequency becomes a fast-varying function. Natural frequency equals the constant value of instantaneous frequency only in linear systems, i.e. when $\dot{\omega} = \dot{A} = 0$ then $\omega_x = \omega$. According to the equation (8) making it possible to determine modal parameters, instantaneous damping factor $h(t)$ is the function of the first two derivatives – signal envelope \dot{A} and instantaneous natural frequency ω . Therefore the statement that changes in the envelope alone influence the damping function is incomplete and may lead to erroneous conclusions.

2.3. Identification of real system

The research into dynamic properties was carried out for the laboratory physical model [14] of transmission tower construction (Fig. 2). The model is made of steel bars 25×3 mm used as rod elements and angles $25 \times 25 \times 2$ mm screwed in the corners of the structure. In the course of the carried out experiments the system was supported with clamps and excited to vibrations with the application of impulse excitation.



Fig. 2. Laboratory physical model of the transmission tower construction

System responses were measured by means of two uniaxial accelerometers, for the purposes of data acquisition 12-bit multichannel measurement card was used. The signals were sampled at 1000 Hz.

Discussed laboratory model makes it possible to analyze dynamic behaviour of the system with various structural damages. By removing individual truss rods, dismantling node connections, the loss of system rigidity and the presence of other defects can be simulated. This reflects a situation when the truss nodes (welds, screw joints) of real object are degraded, which leads to reduction in connection stiffness and resulting loss of the entire segment rigidity. Identification of model parameters was carried out for two system states: undamaged and damaged. In the course of the carried out research the damage was simulated by removing two rods from the lower segment of the transmission tower construction (Fig. 3).

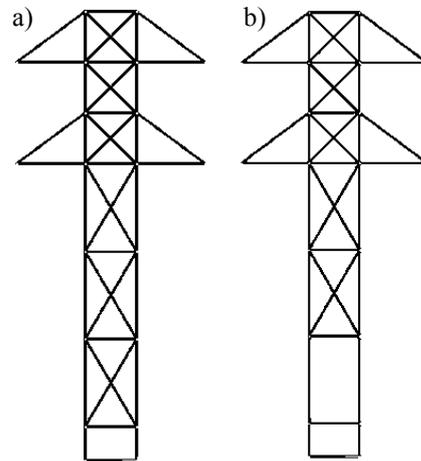


Fig. 3. Schematic diagram of the intact (undamaged) (a) and damaged structure (b)

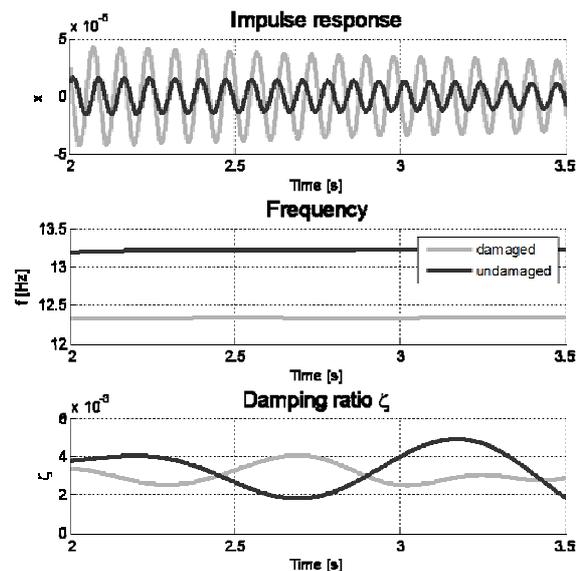


Fig. 4. Identification results for the first mode shape of the transmission tower physical model

Identification results for the first mode shape (Fig. 4) indicate that the natural frequency has changed by 6%. Therefore it can be stated that the loss of two elements has not resulted in the technically significant reduction of the system global rigidity. Damping estimated for the first mode shape has not changed with respect to the undamaged state, which can be explained by the fact that for the considered mode shape dissipative mechanisms are negligible.

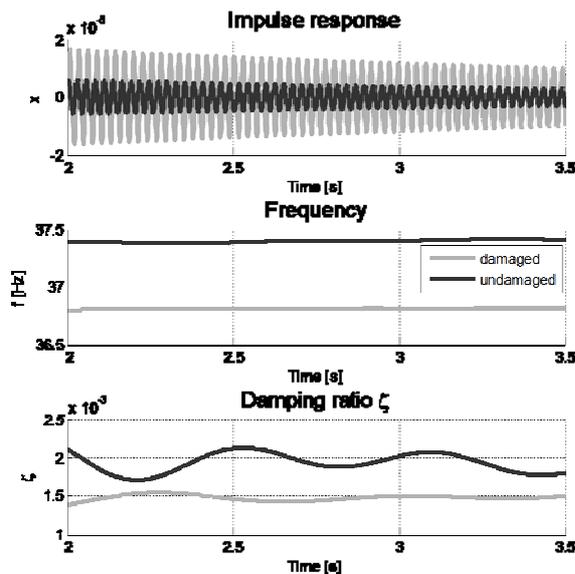


Fig. 5. Identification results for the second mode shape of the transmission tower physical model

Results obtained for the second mode shape (Fig. 5) have revealed small (1%) change in the natural frequency, while system damping has been reduced by approximately 25% with respect to the undamaged state.

3. SUMMARY

The paper presents theoretical background of the nonlinear mechanical systems identification based on Hilbert transform. Described approach was used to investigate dynamic responses of the laboratory model of the real transmission tower. Laboratory setup allowed to induce damages which may occur under operational conditions of the real structures. Based on impulse responses, modal parameters of the laboratory model (natural frequencies and damping factors) for both damaged and undamaged state, were identified. Performed tests indicate that changes in modal parameters resulting from element loss can be easily observed. Nevertheless, it should be noted that damping in this type of constructions substantially results from friction in truss nodes and, therefore, is strictly related to the particular mode shape. The first mode shape of the considered structure is similar to the first mode shape of cantilever beam and, what follows, the relative motion in the truss nodes is

also negligible. Due to this fact, changes in damping in the first mode shape resulting from removal of particular model elements are insignificant and it can be assumed that damping remains constant. Different behavior can be observed for the second mode shape, which is much more complex and therefore the change in damping factor due to the lack of two construction elements is significant (about 25%). Presented results indicate that assumed damage identification procedure based on relatively simple to determine modal parameters combined with simple measurement system can be efficient. Moreover, due to fact that changes in the identified values of modal parameters are significant, they can be used as a damage indicator in other damage identification methods.

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