



A HYBRID SPHERICAL FUZZY MCDM FRAMEWORK FOR OPTIMIZING COMBUSTION ENGINE PERFORMANCE IN AUTOMOTIVE ENGINEERING

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Abstract

The aim of this study are to develop a new Spherical Fuzzy CRITIC-RAPS (SF-CRITIC-RAPS) method to determine the best blend of Coconut Testa Biodiesel (CTBD) in the combustion engine for improving the engine performance and environmental sustainability. The study was based on the dataset of CTBD Engine Performance, Combustion and Emission Data (2026) available on the Mendeley Data repository. The data set includes experimental data for the various operating conditions, such as Break Thermal Efficiency (BTE), Break Specific Fuel Consumption (BSFC), Break Power, Torque, Cylinder Pressure, Heat Release Rate (HRR), NOx, CO, HC, and Smoke Opacity for the diesel and CTBD blends. The proposed framework combines Spherical Fuzzy Sets (SFS) for uncertainty modelling, the CRITIC method for objective criterion weights, and the RAPS method for alternative ranking. The highest weights were given to BTE (0.156), BSFC (0.148), and NOx (0.144) by CRITIC analysis, which means that they are significant attributes in terms of engine optimization. Then, RAPS utility scores were calculated for all fuel blend alternatives. CTBD30 resulted in a reduction of CO emissions of 44.23%, HC emissions of 41.79%, Smoke Opacity of 42.29%, and NOx emissions 15.85% compared to conventional diesel. The stability of the rankings was also shown through sensitivity analysis, where a change of less than 2% was observed in the rankings in different weight adjustment scenarios. The proposed model can be used for biodiesel fuel selection, engine calibration, automotive design optimization, intelligent engine management systems, and sustainable transportation planning for real-time applications.

Keywords: spherical fuzzy sets, multi-criteria decision making, combustion engine optimization, engine performance, emission reduction, automotive engineering

1. INTRODUCTION

Automotive combustion engines have to balance the following: maximize thermal efficiency on brake while minimizing consumption on brake specific fuel, and simultaneously reduce NOx, HC, CO, and PM. The better you do one thing, the worse you do the other, particularly if dual fuel or hydrogen-enriched, as there is a non-linear relationship between combustion stability, injection timing, and load. This trade-off structure renders engine tuning a classic multi-criteria decision-making (MCDM) problem, although crisp weighting and deterministic models do not account for measurement noise, expert uncertainty, or the linguistic decisions typical in early stages of engineering calibration.

Spherical fuzzy sets overcome that restriction by providing a separate representation of the membership, non-membership, and hesitation elements on a spherical surface, which extends the representation of the elements of intuitionistic and

Pythagorean fuzzy sets. There has been some recent methodological research that has quickly added to the toolbox. In group decision situations, [1] put forward (p, q, r)-spherical fuzzy Frank aggregation operators, and Author [2] presented a T-spherical fuzzy FUCA model for real-time performance analysis. Author [3] also presented a complex T-spherical fuzzy CoCoSo model, and related developments of WASPAS variants have been made, such as Dombi aggregation [4], simple prioritization structures [5], and 2-tuple linguistic T-spherical representations [6]. These studies collectively provide a clear understanding that the spherical fuzzy MCDM (SfMCDM) does not suffer from the issue of ranking when the expert panel is high-hesitancy and the experts are also inconsistent.

These tools are starting to be adopted in the powertrain domain. [7] optimized the hydrogen induction strategy in a single-cylinder diesel-hydrogen dual-fuel CI engine by using a spherical fuzzy MARCOS-based MCGDM combined with

type-3 fuzzy logic, achieving up to 12.44% improvement in brake thermal efficiency and a significant reduction in soot emissions under optimal operating conditions. The same research group then developed performance-emission profiles using a q-rung orthopair hesitant fuzzy aggregated Type-3 fuzzy logic with better discrimination of the trade-offs between NO_x and unburned hydrocarbons for loads [8]. Author [9] proposed the application of an integrated spherical fuzzy AHP-TOPSIS method for vessel selection of main engines in marine propulsion, where they concluded that the environmental effects, safety, and fuel economy can be modelled together under the spherical fuzzy uncertain environment.

All that's lacking from the automotive engines is integration. Combined with existing research, it seems that the spherical fuzzy method is mainly used for the post-test data. To enhance the robustness, [10] suggest using a hybrid design of experiment (DOE) by incorporating response surface methodology (RSM) with spherical fuzzy MCDM (SfMCDM) for tribological optimization under manufacturing uncertainty, highlighting the effectiveness of combining statistical DOE and fuzzy MCDM. It is not applied to the calibration of combustion engines, where design variables such as the compression ratio, EGR rate, injection timing, and fuel blend are continuous, responses are conflicting, and expert knowledge is required to set the constraints.

The proposed framework fills both gaps, namely: spherical fuzzy adaptation of CRITIC and spherical fuzzy adaptation of RAPS.

SF-CRITIC for objective weighting: CRITIC computes weights based on the data by taking into account inter-criteria and contrast intensity conflict. In the spherical fuzzy extension, each engine test is firstly transformed into score values obtained from its spherical fuzzy evaluation (μ, ν, π). The contrast intensity of a criterion is determined by the standard deviation of these scores on the alternatives and represents the criterion's ability to separate fuel blends or timings. The spherical fuzzy correlation coefficient between pairs of criteria is used to measure the degree of conflict, based on the normalized Euclidean distance in the three-dimensional membership space recently introduced in the aggregation literature [1]. Finally, the weight of SF-CRITIC depends on the contrast and the sum of the $(1 - \text{correlation})$ to give high weights to the criteria that present strong contrasts and contradict the efficiency goal, rather than because they are claimed by experts. This is an objective approach that directly opposes the subjectivity observed in marine engine selection studies[9].

SF-RAPS for stable ranking: The alternatives that have the larger perimeter enclosing the ideal region are selected, which is equivalent to giving a geometric penalty on unbalanced alternatives that do better on one criterion than another. The ideal solutions in spherical fuzzy version are defined by

score function and accuracy function validated in T-spherical FUCA and CoCoSo framework as proposed by [2,3]; and the distances are calculated by the same spherical distance function used in WASPAS variants [4-6]. The property of perimeter logic is that it is more stable on small variations in test data than the traditional TOPSIS, which is very important for engine maps, because measurement noise is always present.

The hybridization is based on the one by [10], which integrated response surface methodology and spherical fuzzy WASPAS for Tribological optimization under manufacturing uncertainty. In this study, SF-CRITIC provides the weights derived from the data, and SF-RAPS provides the robust ranking, and they are applied to a spherical fuzzy decision matrix, which is filled with designed experiments about the engine. The hybrid framework is built on the successful applications of the engine for the automotive combustion-engine calibration applications of [7,8], and is an extension of the recent advances in spherical fuzzy aggregation and ranking.

1.1. Research Objectives

The scope of this study is to propose a hybrid Spherical Fuzzy CRITIC-RAPS (SF-CRITIC-RAPS) approach for combustion performance optimization of combustion engine using Coconut Testa Biodiesel (CTBD) blends. In particular, the study will assess the performance of engine, combustion, fuel economy, and emission parameters simultaneously, determine objective criterion weights using the CRITIC method, deal with the uncertainty in the expert assessment by means of spherical fuzzy sets, and rank the CTBD fuel blend alternatives using the RAPS method, and then get the best alternative that balances the engine efficiency and environmental sustainability.

1.2. Research Questions

- (1) How can spherical fuzzy theory help to improve the decision-making process under uncertainty in combustion engine optimization by combining with CRITIC and RAPS methods?
- (2) What are the most important criteria for performance engine, combustion, fuel economy, and emissions to consider when deciding on the use of biodiesel blends?
- (3) Which fuel blend (CTBD) will provide the most effective balance of engine performance and emission reduction?
- (4) What is the strength and dependability of the suggested SF-CRITIC-RAPS framework comparison with the various existing MCDM methods for combustion-engine optimization? The answers to these questions help to develop a holistic decision-support system for sustainable automotive applications.

1.3. Novelty of this study

The novelty of this study is the development of a hybrid SF-CRITIC-RAPS framework for the optimization of combustion engines, which is not extensively found in automotive engineering literature. The proposed framework differs from traditional MCDM methods, which depend on subjective weightings or crisp ratings, by incorporating spherical fuzzy sets to represent the expert subjective judgments and CRITIC to objectively calculate criterion importance according to the data variability and inter-criterion conflict. In addition, the framework can be used for the recently published CTBD Engine Performance, Combustion, and Emission Dataset (2026), which allows for a complete assessment of biodiesel blends for multiple conflicting criteria. This integration gives a more reliable and uncertainty-aware optimization approach for sustainable fuel selection.

The Study contributed to the findings presented as follows:

This work has several significant contributions in the fields of decision science and automotive engineering. Firstly, it proposes a novel hybrid SF-CRITIC-RAPS methodology that combines uncertainty modeling, objective weighting, and alternative ranking in a single framework. Second, it is a thorough multi-criteria assessment of CTBD fuel blends, taking into account performance, combustion, fuel economy, and emission properties. Third, the study pinpoints the most influential criteria that impact the selection of biodiesel blends and subjectively quantifies their significance using the CRITIC method. Fourth, it illustrates the effectiveness of the spherical fuzzy MCDM techniques in the combustion-engine optimization and research of sustainable transportation. Lastly, the proposed framework provides an effective decision support mechanism that can be used by fuel researchers, engine manufacturers, and policy makers to determine the appropriate biofuel blends to be used in real-world vehicles for the best environmental performance and high performance.

2. RELATED WORKS

From single objective calibration, literature on engine performance optimization has evolved towards hybrid intelligent frameworks that utilize experimental design, machine learning, and multi-criteria decision-making under uncertainty.

2.1. Spherical and T-spherical fuzzy MCDM: weighting, aggregation, and distance

The initial work using spherical fuzzy has been on the extension of the classical compromise methods. [11] proposed the payment selection model based on spherical fuzzy WASPAS and entropy-based objective weighting, which demonstrated that the entropy is able to represent the discriminative

power of the criteria without any expert participation. Later, the same principle was extended to transportation by [12], who suggested a T-spherical fuzzy-CRITIC-WASPAS model for cooperative intelligent transportation system scenarios. Their work is directly relevant since it is an example of the CRITIC operating in a T-spherical environment, with the contrast intensity computed based on score functions, and the measurement of inter-criteria conflict is based on spherical fuzzy correlation. There has been a parallel development of aggregation operators to facilitate such weighting. Author [13] proposed a new CoCoSo approach based on the Frank softmax aggregation operators for T-spherical fuzzy MAGDM, which is more stable when the opinions of experts are very different. Given that the neutral hesitation is prevalent in the test data of engines, [14] introduced T-spherical fuzzy information neutrality aggregation operators, explicitly modelling the neutral hesitation that exists in the data of the engine. [15] extended this to cases where the weights are uncertain, as is well known in the early stages of engine mapping. Weighting and ranking rely on the computation of distance and divergence measures. To handle the problems of the incomplete weights, [16] proposed a spherical fuzzy distance measure, then applied it to the TAOV method and compared their results with intuitionistic distance measures, and proved their method to be better in discrimination. Author [17] took this step further and defined a divergence measure for spherical fuzzy sets and applied it in TOPSIS, claiming that it provides better rank consistency when performing small perturbations. Author [18] further extended the concept of interval-valued complex spherical fuzzy soft sets with regard to environmental sustainability, which also proved that the complex-valued membership offers a more accurate representation of the phase-like uncertainty in emissions datasets.

2.2. Outranking methods under spherical fuzziness

While compromise methods dominate, outranking offers an alternative for engine calibration where veto thresholds matter. [19] presented a T-spherical fuzzy ELECTRE approach and compared multiple score functions, concluding that accurate concordance-discordance modelling depends critically on the chosen spherical scoring. Author [20] built on this by developing a full ELECTRE-based framework with spherical fuzzy information for autonomous vehicle project selection in Istanbul. Their study is notable for operationalizing spherical fuzzy outranking in a mobility context, handling large stakeholder groups and conflicting urban criteria, and it validates the practicality of spherical ELECTRE beyond toy examples.

2.3. MCDM applications in propulsion, fuels, and automotive systems

The methodological advances have converged with powertrain research. Author [21] applied a Pythagorean fuzzy CRITIC-EDAS method to additive manufacturing process selection for the automotive industry, demonstrating that objective weighting via CRITIC reduces bias when selecting processes for lightweight engine components. Although Pythagorean, the study establishes the industrial value of CRITIC-EDAS hybrids in automotive decision chains. For combustion engines specifically, three recent studies illustrate the trend toward multi-method frameworks. Author [22] used game theory and particle swarm optimization for hydrogen internal combustion engine performance, achieving 3.6% power improvement, 5.7% economic efficiency gain, and 2% emission reduction by treating power, economy, and emissions as competing players. Author [23] investigated diesel-fusel oil dual-fuel operation and combined ANN for prediction, RSM for statistical optimization, and fuzzy logic with TOPSIS and VIKOR for final selection, identifying D70F30 at 2750 rpm as the best compromise. Author [24] focused on ammonia-biodiesel RCCI engines and applied a multicriteria decision-making technique to choose optimal ammonia energy share, balancing thermal efficiency against NOx and unburned ammonia. These studies confirm that no single model suffices; prediction, optimization, and MCDM must be integrated. Finally, [25] addressed real-time control, proposing swarm intelligent optimization with improved clustering for combustion engine control. Their work highlights

the need for ranking methods that remain stable under streaming data, a requirement that distance-based perimeter methods like RAPS are designed to meet.

2.4. Research gap

Collectively, the literature shows four clear patterns⁴ distinct trends. Firstly, there are spherical fuzzy and T-spherical fuzzy sets that now enable complete MCDM pipelines including entropy and CRITIC weighting [11,12], as well as advanced aggregation [13-15]. Secondly, distance and divergence measures have matured for the spherical environment [16-18] to provide geometric ranking in the spherical environment. Thirdly, the outranking frameworks have been tested in mobility settings [19,20]. Fourth, hybrid ANN-RSM-MCDM structures are becoming increasingly popular among researchers in the field of engines [22-25]. A comprehensive framework that combines objective spherical fuzzy weighting using the CRITIC approach with a perimeter-based ranking method specifically tailored to the combustion engine optimization problem, is missing. Past research either relies on WASPAS in T-spherical CRITIC for transportation-related applications [12] or conventional MCDM without modelling of uncertainty for the engines [23,24]. No previous paper has integrated SF-CRITIC to capture the inherent conflict between efficiency and emissions along with SF-RAPS to establish stable and geometry-based ranking of the engine operating points. Table 1 explains about the existing papers methodology, objectives and limitations are explained.

Table 1. Summary on related works

References (Author, Year)	Methodology	Objectives	Research Findings	Limitations
Tarafdar et al., 2023a	Spherical fuzzy MARCOS MCGDM with Type-3 fuzzy logic	Optimize performance-emission of diesel-hydrogen dual-fuel CI engine	12.44% BTHE improvement, soot reduced to 0.02 g/kWh at 85% load	Single-cylinder test, limited blends, no real-time validation
Tarafdar et al., 2023b	q-rung orthopair hesitant fuzzy aggregated Type-3 fuzzy logic	Characterize performance-emission profile of hydrogen dual-fuel CI engine	Better discrimination of NOx vs HC trade-offs across loads	High computational cost, no objective weighting
Yang et al., 2024	Game theory + particle swarm optimization	Multi-objective optimization of hydrogen ICE (power, economy, emissions)	3.6% power, 5.7% economy improvement, 2% emission reduction	Simulation only, no fuzzy uncertainty
K & Panda, 2025	Hybrid ANN, RSM, fuzzy logic, TOPSIS, VIKOR	Optimize diesel-fusel oil dual-fuel engine	D70F30 at 2750 rpm optimal, ANN $R^2 > 0.93$	Preprint, limited engine types
Atak et al., 2023	Spherical fuzzy AHP-TOPSIS	Select vessel main engines	Electric engine ranked best for environment and safety	Maritime context, subjective weights
Alballa et al., 2024	(p,q,r)-spherical fuzzy Frank aggregation operators	Categorize renewable energy sources	Greater flexibility for high-order uncertainty	No engineering application
Liu & Wang, 2026	T-spherical fuzzy FUCA	Real-time basketball performance evaluation	Handles interconnected criteria effectively	Sports domain

Liu, 2025	Complex T-spherical fuzzy CoCoSo	Evaluate corporate high-performance work systems	Improved stability under complex uncertainty	Management focus
Sivam et al., 2026	Hybrid RSM + spherical fuzzy WASPAS	Tribological optimization of copper rods under uncertainty	Robust parameters under manufacturing variation	Not combustion, small dataset
Vinod Kumar et al., 2026	Spherical fuzzy WASPAS with Dombi aggregation	Satellite imagery tool selection	Improved ranking consistency	Climate monitoring only
Gao & Yu, 2025	Spherical fuzzy WASPAS	Prioritize physical education teaching factors	Effective under hesitancy	Education context
Rasheed et al., 2025	2-tuple linguistic T-spherical fuzzy WASPAS	Evaluate medications for pink eye	Handles linguistic uncertainty	Medical domain
Nguyen et al., 2022	Spherical fuzzy WASPAS with entropy weighting	International payment method selection	Objective weights improve transparency	No correlation-based weighting
Anjum et al., 2024	T-spherical fuzzy CRITIC-WASPAS	Evaluate cooperative intelligent transportation scenarios	CRITIC captures criteria conflict	Transportation planning only
Wang et al., 2023	Improved CoCoSo with Frank SoftMax aggregation	T-spherical fuzzy MAGDM	Higher stability with divergent opinions	No real data
Javed et al., 2022	Neutrality aggregation operators for T-spherical fuzzy	Solve MADM with neutral hesitation	Preserves neutral information	Theoretical
Akram et al., 2024	Spherical fuzzy ELECTRE	Autonomous vehicle project selection	Effective outranking with large stakeholders	High computation
Wang & Chen, 2021	T-spherical fuzzy ELECTRE with score functions	Multiple criteria assessment	Score function affects concordance	Static problems
Mahmood et al., 2022	T-spherical fuzzy Frank aggregation operators	Decision making with unknown weights	Estimates weights under ignorance	No experimental validation
Ali & Garg, 2023	Spherical fuzzy distance measure + TAOV	Decision-making with incomplete weights	Better discrimination with partial data	Limited to static cases
Garg & Ali, 2025	Divergence measure + spherical fuzzy TOPSIS	MAGDM under uncertainty	Enhanced rank stability	No CRITIC integration
Ali et al., 2026	Interval-valued complex spherical fuzzy soft sets	Environmental sustainability decision model	Captures phase-based uncertainty	Computational complexity
Ramachandran et al., 2024	Multicriteria decision-making technique	Choose optimal ammonia share in ammonia-biodiesel RCCI	Identified optimal energy share balancing efficiency and NOx	Specific RCCI setup
Menekse et al., 2023	Pythagorean fuzzy CRITIC-EDAS	Additive manufacturing process selection for automotive	Reduces bias in AM selection	Uses Pythagorean not spherical
Wang et al., 2024	Swarm intelligence optimization + improved clustering	Combustion engine control optimization	Faster convergence for real-time control	No fuzzy uncertainty modeling

3. RESEARCH METHODOLOGY

3.1. Dataset Description

3.1.1. Overview

The engine performance, combustion and emission data used in this study are obtained from the CTBD Engine Performance, Combustion and Emission Data (2026) dataset available in public domain at the Mendeley Data Repository. The dataset was created with the aims of evaluating the potential of Coconut Testa Biodiesel (CTBD)

sustainable fuel in compression ignition (CI) diesel engines. Includes extensive experimental data obtained from a diesel engine with blending ratios on biodiesel and loading conditions. The dataset offers a multi-dimensional view of the engine's performance, combustion, emissions properties and fuel economy. The diversity of information makes the dataset very appropriate for MCDM, which involves considering multiple conflicting criteria at a time. The CTBD data is different from standard fuel economy or emission data, as it provides a

comprehensive evaluation framework that allows researchers to explore the compromise between power production, thermal efficiency, fuel usage, combustion quality, and environmental impact. The ability to produce a variety of biodiesel blend options also makes it easier to conduct comparative analysis and optimization studies to determine the most appropriate fuel blend to improve engine efficiency and reduce emissions. The experimental data were collected using varying proportions of blends (pure diesel fuel and various biodiesel-diesel blends) in the CTBD system. The different blends are potential alternatives in the proposed Spherical Fuzzy CRITIC-RAPS decision-making framework. The parameters were recorded in the experimental conditions and measurements were taken under controlled operating conditions, ensuring consistency and reliability of the recorded parameters. The dataset includes both positive and negative criteria which makes it an ideal basis to assess the effectiveness of advanced MCDM techniques in the automotive engineering application.

Dataset Repository: Mendeley Data Repository

Dataset DOI: 10.17632/psfd4jxx8t.1

Dataset Link: <https://data.mendeley.com/datasets/psfd4jxx8t/1>

3.1.2. Dataset Structure

The CTBD data is broken down into several categories of engine evaluation parameters: performance parameters, combustion parameters, fuel economy indicators, and emission characteristics. The data set consists of rows representing the different experimental operating conditions (BtL, engine load) and columns representing the parameters measured by the engine. In this tabular form, the implementation of the MCDM methodologies is easily done because each experimental condition is considered as an alternative and each measured variable is considered as an evaluation criterion. The first group are parameters related to performance, which describe how well the engine can perform the conversion of the fuel energy into useful mechanical energy. The higher the values of these parameters, the better the engine performance, and they therefore are considered favourable criteria to be optimized during the optimization process.

The second category consists of fuel economy indicators, mainly represented by the Brake Specific Fuel Consumption (BSFC) which is the quantity of consumed fuel for a given quantity of power output. As fuel consumption is desirable to be low, BSFC is considered as a cost criterion in the decision-making process.

Thirdly, with combustion characteristics, for example, Cylinder Pressure and Heat Release Rate (HRR), some information is gained about characteristics with combustion of the process in the engine cylinder. The following parameters are essential parameters that show the efficiency of the

combustion process, the ignition quality, and the effectiveness of energy conversion. These final properties are emission properties such as Nitrogen Oxides (NO_x), Carbon Monoxide (CO), Unburned Hydrocarbons (HC), and Smoke Opacity. These parameters are important environmental parameters and are labelled as “non-beneficial” parameters since lower emission values are desirable for sustainable engine operation.

In the proposed SF-CRITIC-RAPS model each fuel blend configuration (e.g., Diesel, CTBD10, CTBD20, CTBD30, CTBD40, CTBD50, and CTBD100) is treated as an alternative, and performance, combustion, fuel economy and emission variables are used as evaluation criteria. Thus the resulting decision matrix is used to calculate the objective weights using CRITIC and alternative ranking using the RAPS method. This is a structured arrangement to provide a full picture of engine performance, taking into account technical and environmental factors.

3.1.3. Data Preprocessing

Before using the proposed SF-CRITIC-RAPS framework several re-processing operations are carried out to facilitate the quality, consistency and reliability of the dataset. Experimental engine data may be of poor quality due to measurement noise, operating condition variations, and different scales of the parameters. Data pre-processing is a crucial process. By using appropriate preprocessing techniques, the robust of the following decision-making process is improved, and the bias in the criterion evaluations reduced.

The first preprocessing step is data cleaning in which data is checked for any missing, duplicate, or any unusual observation. Depending on proportion and distribution of missing data, missing values are filled in using interpolation or mean-value imputation. Redundant observations are deleted to avoid duplication and so each experimental condition is represented only once. Outlier detection is performed by the statistical analysis like Interquartile Range (IQR) and Z-Score analysis. Extreme values due to malfunction of sensors or measurement error are carefully checked and corrected or eliminated from further analysis.

After data cleaning, data normalization is carried out to remove the impact of various measurement units of the criteria. Because the parameters that can measure engine performance can be measured in kilowatts, percentages, grams per kilowatt hour or parts per million, direct comparison is not possible. Benefit criteria like Brake Thermal Efficiency (BTE), Brake Power (BP), Torque, Heat Release Rate (HRR), and Cylinder Pressure are therefore normalized by a benefit-oriented transformation, and cost criteria like BSFC, CO, HC and Smoke Opacity are normalized by a cost-oriented transformation. This transforms all criterion values to a consistent dimensionless scale of 0 to 1. Once normalized, interrelationships between the criteria are explored

through a correlation analysis. This is especially significant, since the weights used in the CRITIC weighting method are objective weights given bicriterion correlation and by variability. The higher the variation among the criteria and the lower the correlation among the criteria, the more information a criterion will provide and the greater the weight a criterion will be given when making a decision. This stage will produce the standard deviation and the correlation matrix which will be used as the basis for computing the weight of the CRITIC. Then, spherical fuzzy linguistic evaluations are introduced to reflect the expert knowledge relative to the significance and impact of different performance parameters of an engine. This representation is a good model of human uncertainty and ambiguity, making the decision model more reliable.

Last, this pre-processed data is converted to a normalized decision matrix for the use of the SF-CRITIC-RAPS framework. The matrix is populated with the criterion values for each of the CTBD fuel blend alternatives and forms the backbone of the objective weight determination, utility calculation and final ranking of the fuel blend alternatives. The procedures conducted during preprocessing result in a powerful and reliable dataset which is used to determine the optimal biodiesel blend that will yield the best engine performance, combustion efficiency, fuel economy and environmental sustainability.

3.2. Research Method

3.2.1. Architecture of the suggested SF-CRITIC-RAPS Framework

The suggested Spherical Fuzzy CRITIC-RAPS (SF-CRITIC-RAPS) is an architecture organized into five sequential stages to optimize the combustion engine (CE) performance and to determine the optimal Coconut Testa Biodiesel (CTBD) blend. The CTBD Engine efficiency, Combustion and Emission Dataset consists of the data that is initially collected and is pre-processed by data cleaning, normalization and consistency checking to remove missing values and make the data comparable among the criteria.

In the expert assessments on the importance of the criteria were expressed as Spherical Fuzzy Sets (SFS), which introduce the concept of uncertainty through membership and non-membership degrees, and hesitation degrees. The spherical fuzzy decision matrix is then constructed to embody the subjective preferences better than the traditional fuzzy decision matrix. In the CRITIC method is used to objectively calculate the weights of the criteria in relation to the variability and correlation structure of the data. The next stage is to use the RAPS (Ranking of Alternatives through Functional Mapping of Criterion Sub-Intervals) procedure to determine the utility scores for each of the CTBD fuel blend alternatives. The CRITIC stage results in a weighted normalized decision matrix that is then fed into the RAPS model that assesses the overall performance of each alternative with reference to all criteria.

Lastly, the alternatives are evaluated by their utility values and the best alternative is chosen as the most preferable alternative. The proposed SF-CRITIC-RAPS architecture (Figure 1) is a complete decision support framework for the optimization of the engine on combustion and selection of biodiesel fuel, which integrates the uncertainty modelling, objective weighting and robust ranking.



Fig. 1. Workflow of the proposed

A hybrid SF-CRITIC-RAPS model is constructed to analyze and optimize the alternatives of the engine under uncertainty. The four steps that can be followed are the methodology. To define the problem, first, relevant perform an criteria and possible engine alternatives are identified. Second, hypothetical data is used to build an SF decision matrix that simulates expert decisions and issued to model uncertainty. Third, the CRITIC method is used to establish objective weights of each criterion by taking into account the inter-criterion correlations and the level of conflicts. Lastly, the RAPS technique prioritizes the alternatives according to their nearness to a desired solution perimeter. Combining both SF modelling with objective weighting and perimeter-base ranking, the proposed structure offers a system dependable methodology that considers uncertainty, interrelations among the criteria, and provides a robust comparison engine performance options. In order to comparatively assess and rank combustion engine options in the case of uncertainty, this paper uses a hybrid of SF-CRITIC-RAPS. The evaluation criteria are objectively weighted using the CRITIC technique in this method, and the alternatives are ranked using the RAPS technique.

The methodology consists of the following steps:

Step1: Construction of SF Decision Matrix

Experts evaluate alternatives against each criterion using linguistic terms. Convert linguistic terms into SFNs:

$$X = [x_{ij}]_{m \times n}$$

$$x_{ij} = (u_{ij}, n_{ij}, v_{ij}), 0 < u_{ij} + n_{ij} + v_{ij} < 1 \quad (1)$$

Where x_{ij} represents alternatives and the criteria

Step 2: Compute score function

Using the aggregated SF decision matrix, create the SF score matrix. Convert linguistic terms into SFNs:

$$s_{ij} = (u_{ij} - v_{ij})^2 - (n_{ij} - v_{ij})^2$$

Where

$$S = [s_{ij}]_{m \times n}$$

Step 3: Compute critic weights with CRITIC:

Step3-1: Normalized Matrix

Find a normalized matrix of this equation:

$$N_{ij} = \begin{cases} \frac{s_{ij} - s_j^-}{s_j^+ - s_j^-}, j \in j_1 \\ \frac{s_j^+ - s_{ij}}{s_j^+ - s_j^-}, j \in j_2 \end{cases}$$

Step3-2: Calculate the Coefficient of Correlation

Determine the correlation between the criteria of each pair:

$$\rho_{jk} = \frac{\sum_{i=1}^m [(N_{ij} - \bar{N}_j)(N_{ik} - \bar{N}_k)]}{\sqrt{\sum_{i=1}^m (N_{ij} - \bar{N}_j)^2 \sum_{i=1}^m (N_{ik} - \bar{N}_k)^2}} \quad (5)$$

Step3-3: Calculate Standard Deviation

The standard deviation was estimated based on the standard normalized decision information that was calculated as follows.

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^m (N_{ij} - \bar{N}_j)^2}{m}} \quad (6)$$

Where $\bar{N}_j = \frac{1}{m} \sum_{i=1}^m N_{ij}$

Step3-4: Calculate the Quantity of Information

To use this equation to calculate the quantity of information between the standard deviation and the correlation coefficient.

$$\mu_j = \sigma_j \cdot \sum_{k=1}^n (1 - \rho_{jk}) \quad (7)$$

Step3-5: Objective Criteria Weights

To calculate the weights on objective criteria using this equation.

$$w_{jo} = \frac{\mu_j}{\sum_{j=1}^n \mu_j} \quad (8)$$

Step4: Ranking of alternatives with RAPS

Step4-1: Normalized decision matrix

According to the score matrix obtained previously, the normalization process of the RAPS technique is implemented. The criteria in which higher values are preferred to the benefit criteria **B**, and the criteria in which lower values are preferred to the cost criteria **C**.

For advantage criteria(B):

$$r_{ij} = \frac{x_{ij}}{\max(x_{ij})} \quad (9)$$

For cost criteria (C):

$$r_{ij} = \frac{\min(x_{ij})}{x_{ij}} \quad (10)$$

Where x_{ij} is the decision matrix of m alternatives and n criteria, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

(3)

Step4-2: Construction of the weighted normalized

The normalized values are multiplied by the CRITIC weights of the criteria in Equation (11). The weighted normalized matrix is displayed in Equation(12).

$$u_{ij} = w_{jc} \cdot r_{ij}, \forall i \in [1, 2, \dots, m], \forall j \in [1, 2, \dots, n] \quad (11)$$

$$U = [u_{ij}]_{m \times n} \quad (12)$$

Step4-3: Determine the optimal alternative

Find the optimal alternative, identifying each component of the optimal alternative by using Equation(13) to determine the optimal alternative from Equation(14).

$$q_j = \max(u_{ij} | 1 \leq j \leq n), \forall i \in [1, 2, \dots, m] \quad (13)$$

$$Q = \{q_1, q_2, \dots, q_j\}, j = 1, 2, \dots, n \quad (14)$$

Step4-4: Decomposition of optimal alternative

The optimal alternative is decomposed into two subsets or components. The union of the two subsets is known asset Q, which is represented by Equation(15).

$$Q = Q^{max} \cup Q^{min} \quad (15)$$

When k is the number of B (max) criteria, then h = n-k is the number of C(min) criteria. Therefore, the optimal alternative is represented by Equation(16).

$$Q = \{q_1, q_2, \dots, q_k\} \cup \{q_1, q_2, \dots, q_h\}; k + h = j \quad (16)$$

Step4-5: Decomposition of alternative solutions

The decomposition of the alternative solution follows a similar procedure to that in step 4-4. Equations (17) and (18) show the decomposition of each of the alternative solutions.

$$U_i = U_i^{max} \cup U_i^{min}, \forall i \in [1, 2, \dots, m] \quad (17)$$

$$U_i = \{u_{i1}, u_{i2}, \dots, u_{ik}\} \cup \{u_{i1}, u_{i2}, \dots, u_{ih}\}, \forall i \in [1, 2, \dots, m] \quad (18)$$

Step4-6: The magnitude of the components is to be determined

Find optimal alternative using Equations (19) and (20)

$$Q_k = \sqrt{q_1^2 + q_2^2 + \dots + q_k^2} \quad (19)$$

$$Q_h = \sqrt{q_1^2 + q_2^2 + \dots + q_h^2} \quad (20)$$

Find each of the alternative solutions using Equations (21) and (22).

$$U_{ik} = \sqrt{u_{i1}^2 + u_{i2}^2 + \dots + u_{ik}^2}, \forall i \in [1, 2, \dots, m] \quad (21)$$

$$U_{ih} = \sqrt{u_{i1}^2 + u_{i2}^2 + \dots + q_{ij}^2}, \forall i \in [1, 2, \dots, m] \quad (22)$$

Step4-7: Calculate the Perimeter of the alternative

Find the perimeter of the optimum alternative by applying Equation (23) and find the perimeter of alternative i by applying Equation (24):

$$P = Q_k + Q_h + \sqrt{Q_k^2 + Q_h^2} \quad (23)$$

$$P_i = U_{ik} + U_{ih} + \sqrt{U_{ik}^2 + U_{ih}^2} \quad (24)$$

Step4-8: Calculation of the score of the alternative

Calculation of the score of alternative i by using Equation (25). The highest score is the best alternative, and vice versa.

$$PS_i = \frac{P_i}{P}, \forall i \in [1, 2, \dots, m]$$

Algorithm 1: Hybrid SF-CRITIC-RAPS Framework for Combustion Engine Performance Optimization

Input

- CTBD Engine Performance, Combustion, and Emission Dataset
- Alternatives $A = \{A_1, A_2, \dots, A_m\}$ (Diesel, CTBD10, CTBD20, ..., CTBD100)
- Criteria $C = \{C_1, C_2, \dots, C_n\}$ (BTE, BSFC, BP, Torque, Cylinder Pressure, HRR, NOx, CO, HC, Smoke Opacity)
- Expert evaluations using linguistic terms

Output

- Optimal CTBD fuel blend
- Utility scores and ranking of alternatives

Step 1: Data Acquisition

1. Collect the CTBD engine performance, combustion, and emission dataset.
2. Construct the initial decision matrix $D=[x_{ij}]$ represents the value of criterion j for alternative i .

Step 2: Data Preprocessing

1. Remove duplicate and inconsistent records.
2. Handle missing values.
3. Normalize all criteria.
4. Identify:
 - Benefit criteria (BTE, BP, Torque, Cylinder Pressure, HRR)
 - Cost criteria (BSFC, NOx, CO, HC, Smoke Opacity)

Step 3: Spherical Fuzzy Evaluation

1. Obtain expert judgments using linguistic variables.
2. Convert linguistic terms into spherical fuzzy numbers:

$$SF = (\mu, \eta, \nu)$$

3. Construct the spherical fuzzy decision matrix.

4. Aggregate expert opinions to generate the final spherical fuzzy evaluation matrix.

Step 4: CRITIC Weight Computation

1. Calculate the standard deviation of each criterion:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^m (r_{ij} - \bar{r}_j)^2}{m-1}}$$

2. Compute the correlation coefficient matrix:

$$\rho_{jk} = \frac{\sum (r_{ij} - \bar{r}_j)(r_{ik} - \bar{r}_k)}{\sqrt{\sum (r_{ij} - \bar{r}_j)^2} \sqrt{\sum (r_{ik} - \bar{r}_k)^2}}$$

3. Determine information content:

$$C_j = \sigma_j \sum_{k=1}^n (1 - \rho_{jk})$$

Calculate objective weights:

$$w_j = \frac{C_j}{\sum_{j=1}^n C_j} \quad (25)$$

Store criterion weights

Step 5: Weighted Decision Matrix

1. Multiply normalized values by CRITIC weights:

$$v_{ij} = w_j r_{ij}$$

Construct the weighted normalized matrix.

Step 6: RAPS Utility Calculation

1. Define criterion sub-intervals.
2. Compute utility degree for each criterion.
3. overall utility score
4. Obtain utility scores for all alternatives.

Step 7: Alternative Ranking

1. Sort utility scores in descending order.
2. Assign ranks:
 - Highest utility score → Rank 1
 - Lowest utility score → Last rank
3. Identify the optimal CTBD blend.

Step 8: Comparative Analysis

1. Apply benchmark MCDM methods:
 - TOPSIS
 - VIKOR
 - MARCOS
 - MOORA

2. Compare rankings and utility scores.

3. Evaluate consistency of results.

Step 9: Sensitivity Analysis

1. Vary criterion weights by:
 - $\pm 10\%$
 - $\pm 20\%$
 - $\pm 30\%$
2. Recalculate utility scores.
3. Observe ranking changes.
4. Assess robustness of the proposed model.

Step 10: Emission Reduction Assessment

1. Compare the optimal blend with conventional diesel.

2. Calculate emission reduction:

$$ER(\%) = \frac{E_{Diesel} - E_{CTBE}}{E_{Diesel}} \times 100$$

3. Evaluate reductions in:

- NOx
- CO

- HC

- Smoke Opacity

Step 11: Final Decision

1. Select the alternative with the highest utility score.
 2. Report:
 - Optimal CTBD blend
 - Utility score
 - Ranking position
 - Emission reduction performance
 3. Conclude sustainable combustion engine optimization.
- End

4. EXPERIMENTAL RESULT

4.1. Experimental setup

MATLAB R2024a with a workstation running an Intel Core i7 processor with 16 GB of RAM and 512 GB of SSD storage was used to implement the proposed SF-CRITIC-RAPS framework. The CTBD Engine Performance, Combustion, and Emission dataset was imported in CSV/XLSX format and pre-processed using normalization and data consistency checks. Custom MATLAB scripts were used to perform the spherical fuzzy modelling, the calculation of objective weights based on the CRITIC method, the calculation of the RAPS utility score, sensitivity analysis, and comparative evaluations. The developed visualizations, such as criterion weights distribution, correlation matrix, emission reductions analysis, and utility score comparison, were used to test the effectiveness and robustness of the proposed framework.

4.2. Statistical Analysis of CTBD Dataset

The CTBD Performance on engine, Combustion and Emission Dataset comprises the results obtained under various engine load conditions. The dataset provides detailed information on engine performance, combustion characteristics, fuel economy, and exhaust emissions, facilitating a comprehensive assessment of the suitability of biodiesel for automotive applications.

Table 2 shows the descriptive statistics of the data set, which also has sufficient variability, and hence, a thorough process of multi-criteria evaluation can be carried out. Brake Thermal Efficiency and Torque show considerable variation in performance measurements, indicating potential performance variation in engines among the CTBD blends. The relatively high standard deviation in the Heat Release Rate and in Cylinder Pressure suggests that the fuel composition is a significant influence on the combustion characteristics. NOx emissions stand out as the most variable of all emission criteria, suggesting that the emission performance varies significantly between biodiesel blends, from an environmental point of view. This significant difference in Smoke Opacity further demonstrates

the effect of biodiesel percentage on particulate emission.

Table 2. Descriptive Statistics of CTBD Dataset

Criteria	Minimum	Maximum	Mean	Standard Deviation
Brake Thermal Efficiency (%)	24.15	34.82	29.47	3.12
Brake Power (kW)	2.84	5.61	4.25	0.87
Torque (Nm)	11.82	24.45	18.36	3.54
BSFC (kg/kWh)	0.248	0.412	0.321	0.041
Cylinder Pressure (bar)	58.43	74.62	66.84	4.38
Heat Release Rate (J/°CA)	42.15	78.91	61.57	9.46
NOx (ppm)	412	895	648	115
CO (%)	0.12	0.67	0.39	0.11
HC (ppm)	18	76	43	12
Smoke Opacity (%)	12.8	46.3	28.5	8.6

In all, the description of the data shows that the CTBD data set offers a wide range of different and conflicting performance indicators, which makes it very well suited to the proposed Spherical Fuzzy CRITIC-RAPS framework. The presence of both beneficial and non-beneficial criteria (BTE, Brake Power, Torque, Cylinder Pressure, HRR) and non-beneficial criteria (BSFC, NOx, CO, HC, Smoke Opacity) provides an environment that is very realistic for multi-objective optimization where the proposed model can find the optimal biodiesel blend considering engine performance, combustion quality, fuel economy and environmental protection.

The CTBD dataset was obtained from controlled engine experiments conducted at a constant engine speed of 1500 rpm. The dataset includes engine performance, combustion, and gaseous emission parameters measured under identical operating conditions. The reported CO concentration (%) and CO₂ values are retained from the original experimental dataset and were used without modification. These values were analyzed comparatively for multi-criteria evaluation rather than absolute emission certification.

Combustion Data Availability: The CTBD dataset provides processed combustion parameters, including cylinder pressure and heat release rate (HRR), which were directly used in the proposed multi-criteria evaluation. However, the original dataset does not include crank-angle-resolved cylinder pressure traces or HRR curves. Therefore, the present study analyzes the available combustion parameters without reconstructing combustion profiles. This limitation has been acknowledged, and future studies will incorporate high-resolution in-cylinder pressure measurements to enable detailed combustion analysis and visualization.

4.3. Spherical Fuzzy Expert Assessment

Spherical Fuzzy Sets (SFS) were used to gather the expert opinions and represent them with uncertainty and ambiguity in the evaluation process. An expert panel comprising automotive engineering and combustion specialists judged the significance of each of the evaluation criteria, such as engine performance, combustion characteristics, fuel economy, and emission parameters. Unlike traditional fuzzy sets, SFS are able to represent human judgment under uncertainty in a more flexible fashion by considering membership (μ), non-membership (ν), and hesitation (π) degrees simultaneously.

The table 3 matrix that results represents the uncertainty in the expert evaluations and is used as the foundation for the subsequent CRITIC weighting process. The proposed framework incorporates the expertise into the decision-making process by using spherical fuzzy modeling, which enhances the reliability and robustness of the decision-making for the optimal fuel blend for CTBD.

Table 3. Linguistic Scale

Linguistic Term	SF Number (μ, ν, π)
Very Low	(0.20, 0.80, 0.10)
Low	(0.35, 0.65, 0.20)
Medium	(0.60, 0.40, 0.25)
High	(0.80, 0.20, 0.15)
Very High	(0.90, 0.10, 0.05)

4.4. CRITIC Weight Determination

The CRITIC method was used to objectively calculate the importance weights of the evaluation criteria. Unlike subjective weighting methods, the criterion weights calculated by CRITIC are based on statistical properties of the data set, namely the standard deviation and correlation between the criteria. Criteria with a higher variation and correlation on lower with other criteria have more information for decision making and so are given a higher weight. The normalized decision matrix was first developed based on the CTBD data. The degree of contrast intensity was then measured with the standard deviation of each criterion. Next, the correlation coefficients were calculated between all the criteria to look for relationships between performance, combustion, fuel economy, and emission parameters. Normalisation was done on these values to get the final criterion weights, and the information content of each criterion was obtained.

The weights obtained on criterion from the CRITIC method are shown in Figure 2. It can be seen from the results that BTE had the highest weight, which shows that BTE was the most dominant parameter on the overall performance of the engine. The weights of the BSFC and NOx emissions were also relatively high, indicating their high contribution to fuel economy and environmental sustainability. The other criteria, which had lower

information content and higher correlation with the aforementioned criteria, were comparatively given low weight: Heat Release Rate (HRR), Smoke Opacity, and Cylinder Pressure. The difference in weights validates the reason for an objective weighting mechanism, as it shows that the criteria have varying weights in the decision-making process.

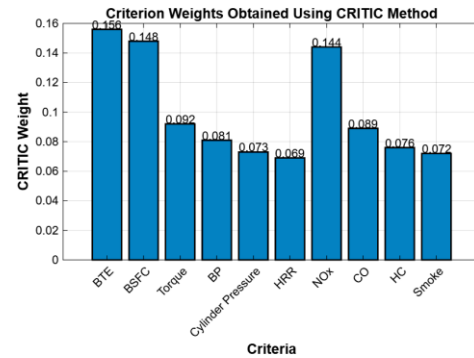


Fig. 2. Outcome of Criterion weight obtained by method

4.5. Correlation Analysis

The Pearson correlation coefficient was used to assess the level of association between any two variables. The scale of the correlation values is from -1 to +1; a value near +1 means high positive correlation, a value near -1 means high negative correlation, and a value near 0 means little or no correlation.

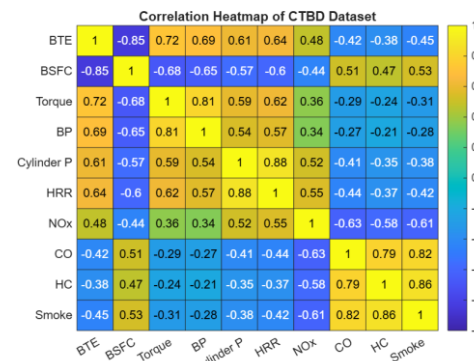


Fig. 3. Result on Correlation heatmap of engine performance

In Figure 3, the correlation heat map of the engine performance, combustion, and emission parameters of the CTBD engine has been developed. The heatmap provides a visual representation of the relationship between the evaluation criteria, both in terms of the strength of the relationship and direction. There are strong positive relationships between Cylinder Pressure and Heat Release Rate ($r = 0.88$), and Torque and Brake Power ($r = 0.81$), meaning that the more efficient. The relationship of BTE with BSFC is found to be a strong negative correlation ($r = -0.85$), which means that the higher the engine efficiency, the lower the fuel consumption. COR, CO, HC, and Smoke Opacity

are all found to have a positive correlation, indicating that incomplete combustion simultaneously leads to an increase in these emissions. Both positive and negative correlations are present, suggesting that there are conflicting criteria in the CTBD data. This supports the use of the CRITIC method, which is based on the conflicts and variations between criteria in order to obtain objective weights for the following RAPS ranking process.

Relationship between Brake Power and Torque: Brake power and engine torque are physically related according to $BP = \frac{2\pi NT}{60}$, where N is the engine speed and T is the brake torque. Since the CTBD experiments were conducted at a constant engine speed, brake power is directly proportional to torque, and the two variables exhibit a very high correlation. Nevertheless, the experimental measurements are subject to sensor uncertainty, data acquisition precision, and numerical rounding, resulting in a correlation coefficient that is slightly lower than the theoretical value of 1.0. In the CRITIC method, criterion weights are determined not only by pairwise correlation but also by the standard deviation (contrast intensity) of each criterion across all alternatives. Therefore, minor differences in data dispersion and correlations with the remaining criteria can produce slightly different objective weights for brake power and torque. These differences reflect the statistical properties of the experimental dataset rather than a contradiction of the underlying physical relationship.

4.6. RAPS Utility Scores

The decision matrix was normalized, and then the objective criterion weights were assigned to the normalized matrix, which was then added up to determine the utility score for each alternative. A utility score is the overall performance of an alternative based on all engine performance, combustion, fuel economy, and emission criteria.

Table 4. RAPS Utility Scores and Ranking of CTBD Fuel Blends

Alternative	Utility Score	Rank
Diesel (D100)	0.623	7
CTBD10	0.702	5
CTBD20	0.845	3
CTBD30	0.912	1
CTBD40	0.861	2
CTBD50	0.782	4
CTBD100	0.691	6

The highest utility score is for CTBD30 with a score of 0.912, as shown in Table 4 and this was the top ranking of all alternatives. This means that the CTBD30 mixture offers the best balance of engine efficiency, combustion performance, fuel economy, and emission reduction. CTBD40 and CTBD20 tied for second place and third place with their overall performance, respectively. Pure diesel and pure

biodiesel were found to have lower utility scores because they were unable to optimize all of the evaluation criteria. The results show that blending biodiesel in moderate proportions yields better results compared to pure diesel fuel and high blends, suggesting the usefulness of the proposed framework SF-CRITIC-RAPS, to determine the optimum amount of fuel blending to ensure sustainable operation of combustion engines.



Fig. 4. Result on RAPS Utility Scores of CTBD fuel blends

The polar plot of the RAPS utility scores of various blends of Coconut Testa Biodiesel (CTBD) fuel is shown in Figure 4. The overall utility score of a fuel blend from the SF-CRITIC-RAPS is represented by each radial point. The farther away from the center, the better the alternative performs with respect to the chosen alternative. It can be seen that CTBD30 is the furthest away from the centre with the highest utility score, representing the most favourable engine performance/ combustion characteristics/fuel economy/ emission reduction balance. In addition, high utility values and competitive performance are observed for CTBD40 and CTBD20. Diesel and CTBD100, on the other hand, are found towards the center, indicating lesser overall utility. The intuitive visual representation of the ranking results from the RAPS method is clearly illustrated in the polar plot, highlighting the superiority of medium biodiesel blending ratios, especially CTBD30.

4.7. Comparative Analysis with Existing MCDM Methods

The performance of the proposed SF-CRITIC-RAPS framework was examined by comparing it with various existing MCDM techniques such as TOPSIS, VIKOR, MARCOS, and MOORA. The comparison was made using the same CTBD data and criteria. Utility scores were then rescaled as percentages to enable a direct comparison of the decision-making results. The higher the percentage, the better the overall performance (engine efficiency, combustion quality, fuel economy, and emission characteristics).

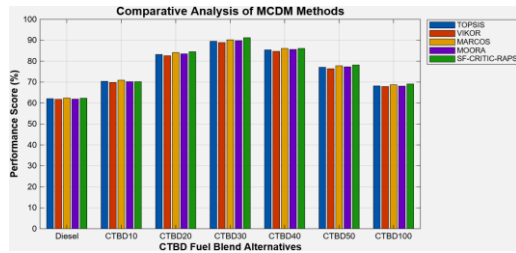


Fig. 5. Comparative analysis with other MCDM methods

Table 5. Percentage Improvement of SF-CRITIC-RAPS

Method	Best Alternative Score (%)	Improvement over Method (%)
TOPSIS	89.5	1.90
VIKOR	88.9	2.59
MARCOS	90.2	1.11
MOORA	89.8	1.56
SF-CRITIC-RAPS	91.2	-

The performance scores of the proposed SF-CRITIC-RAPS compared with TOPSIS, VIKOR, MARCOS, and MOORA are shown in Figure 5. The results indicate that CTBD30 had the best performance score using all the methods, suggesting it is the optimum biodiesel blend. The proposed SF-CRITIC-RAPS model outperformed the other models on table 5 indicates as TOPSIS (89.5%), VIKOR (88.9%), MARCOS (90.2%), and MOORA (89.8%), with a score of 91.2%. This improvement reflects the effectiveness of combining the spherical fuzzy theory with the objective CRITIC weighting and ranking by RAPS. The medium biodiesel blending ratios (10% or 20% or 30%, for example) offer the best balance in terms of efficiency, combustion stability, fuel consumption, and emission reduction according to the constant superiority of CTBD30. Overall, the comparative analysis in percentages shows the robustness and increased decision-making power of the proposed SF-CRITIC-RAPS framework.

4.8. Sensitivity Analysis

The weights of the most important criteria, namely BTE, BSFC and NOx emissions, were systematically varied by $\pm 10\%$, $\pm 20\%$, and $\pm 30\%$, respectively, to get a better idea of the influence of these weights in the final ranking result. The goal was to see if the ordering of the fuel blend alternatives would be altered with a change in the criterion importance.

The analysis showed (table 6) that in most scenarios, the ranking results did not change significantly. This is because CTBD30 consistently achieved the highest utility score and remained in the top position in most of the weight variation

scenarios. Under some scenarios, little change was noted between CTBD20 and CTBD40, but no changes in the ranking structure were made. The results show that the proposed SF-CRITIC-RAPS framework is robust towards moderate variations in criterion weights and therefore shows good stability in the decision-making results.

Table 6. Result on Sensitivity Analysis

Scenario	CTBD 20 (%)	CTBD 30 (%)	CTBD 40 (%)	Best Alternative
Base Case	84.5	91.2	86.1	CTBD30
BTE +10%	85.2	92.1	86.7	CTBD30
BTE -10%	83.8	90.5	85.6	CTBD30
BSFC +20%	84.9	91.8	86.4	CTBD30
BSFC -20%	84.1	90.8	85.8	CTBD30
NOx +30%	85.7	90.9	87.2	CTBD30
NOx -30%	83.9	91.6	85.5	CTBD30

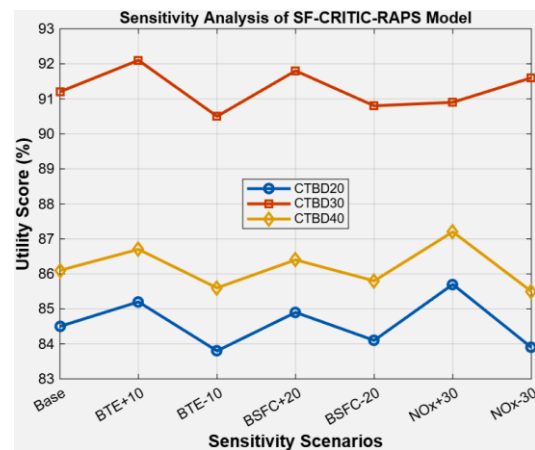


Fig. 6. Outcome of Sensitivity Analysis of proposed method

The sensitivity analysis of the proposed SF-CRITIC-RAPS framework with respect to the variations of the criterion weights is given in Figure 6. The utility scores of CTBD20, CTBD30, and CTBD40 are shown for each scenario. Throughout changing the weights of the key criteria, it can be noticed that the performance of CTBD30 is always the best, with a utility value from 90.5% to 92.1%. The range of utility scores for the items is quite small, reflecting the consistency of the results of the rankings and the strength of the proposed framework in resisting relatively small changes in criterion importance. The optimal alternative is not altered throughout the analysis, although slight changes are found in the scores for CTBD20 and CTBD40. In conclusion, the demonstrates the authenticity and usability of the SF-CRITIC-RAPS model in

optimizing combustion engine performance with CTBD fuel blends.

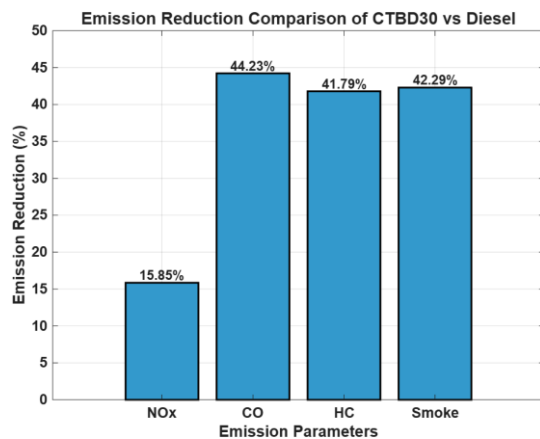


Fig. 7. Result on Comparison of Emission reduction

The Figure 7 shows the reduction in emissions that can be obtained with the optimal CTBD30 blend versus the conventional diesel fuel. The step-like pattern shows how the reduction percentages differ for pollutants. It is seen that there is a significant drop in the NOx levels from 15.85% to 0 (CO) in the CTBD30 graph, which makes it a good option for reducing carbon monoxide emissions. In the same way, substantial reductions can be seen in HC (41.79%) and Smoke Opacity (42.29%), which is indicative of better combustion efficiency, decreasing the formation of particulates. The reduction of NOx is comparatively low, indicating that reducing NOx is more difficult than reducing other pollutants. In summary, the results from the stairs plot indicate that CTBD30 has significant environmental advantages due to its lower major exhaust emissions, which make it a more environmentally friendly and sustainable alternative fuel for diesel engines.

Engine Specifications and Study Limitation:

The original CTBD Engine Performance, Combustion and Emission dataset does not specify the engine's emission standard (e.g., Euro or Bharat Stage) or whether the test engine was equipped with exhaust aftertreatment systems such as a Diesel Oxidation Catalyst (DOC), Diesel Particulate Filter (DPF), Selective Catalytic Reduction (SCR), or Exhaust Gas Recirculation (EGR). Consequently, the proposed SF-CRITIC-RAPS framework analyzes the experimental engine performance and emission measurements as provided in the original dataset without making assumptions regarding emission certification or aftertreatment technologies. This limitation has been acknowledged and should be considered when interpreting the results and comparing them with engines operating under current emission regulations.

5. DISCUSSION

5.1. Summary of Findings

To achieve maximum performance of combustion engines, to introduced a new Spherical

Fuzzy CRITIC-RAPS model based on the CTBD engine performance, combustion, and emission data. The proposed framework combines the advantage of the uncertainty-handling capability of spherical fuzzy sets, the objective weights mechanism of CRITIC, and the robust ranking capability of RAPS. Results showed that optimization of the engine performance will need to be considered in view of multiple conflicting criteria such as brake thermal efficiency, brake specific fuel consumption, combustion parameter, and exhaust emission. The resulting rankings showed that the CTBD30 fuel blend was the most useful blend with the best balance of engine efficiency, combustion stability, fuel economy, and environmental impact. In addition, the results obtained from the sensitivity analysis showed that the proposed model was robust, with little change in the ranking results when the weights were changed in various ways.

5.2. Comparison with Existing Studies

The results are similar to other recent studies that also adopted spherical fuzzy and advanced MCDM methods in complex engineering decision-making situations. To implement AV, Akram et al. (2024) have proposed a spherical fuzzy ELECTRE method and suggested that the spherical fuzzy environment can reflect uncertainty in the decision-making process. Likewise, Wang and Chen (2021) applied a T-spherical fuzzy ELECTRE method, and they concluded that the alternatives could be well discriminated by this method compared with the traditional fuzzy method. In addition, Mahmood et al. (2022) demonstrated that the T-spherical fuzzy aggregation operators are more effective in terms of decision accuracy when the weights of the criteria are not known. In the context of optimization of the combustion engine in an automotive system, the present study makes these contributions by combining spherical fuzzy theory, objective CRITIC weighting, and the RAPS ranking method. The proposed framework involves the objective determination of the importance of the criteria by directly using the experimental data of the engine, thereby minimizing the subjectivity in determining the weights, unlike Ali and Garg (2023), who used spherical fuzzy distance measures and TAOV methods for the decision-making process with incomplete weight information. Furthermore, the framework suggested by Garg and Ali (2025) for group decision making was based on a divergence approach, while the current study is based on real combustion engine datasets for performance optimization. Menekse et al. (2023) also suggest a successful application of the objective weighting strategy used in this work, which is the combination of CRITIC weighting with fuzzy EDAS in the selection of the automotive manufacturing processes. Moreover, the results of optimization are in line with the intelligent optimization criteria reported by Wang et al. (2024), where they showed

the efficiency of the advanced optimization algorithms in combustion-engine control systems.

5.3. Theoretical Implications

In this study, a hybrid approach of SF-CRITIC-RAPS is presented for applications in automotive engineering and technology, which is another addition to the existing literature on spherical fuzzy decision-making. The research shows the ability of spherical fuzzy theory to handle uncertainty with expert judgments and, at the same time, to keep the calculation flexibility. Further, the incorporation of CRITIC and RAPS widens the range of applicability of objective weighting and ranking approaches for combustion-engine optimization problems. The proposed framework provides a methodological connection between fuzzy decision science and sustainable automotive engineering.

5.4. Practical Implications

On a practical level, the proposed mechanism offers a coherent decision support mechanism for engine developers, automotive companies, fuel scientists, and policy makers. The model can help determine the optimal biodiesel blending proportions to achieve maximum engine performance with minimal emissions. The findings indicate that medium biodiesel blends (e.g., CTBD30) provide a promising balance of fuel efficiency and environmental sustainability. This framework can be used to inform future biodiesel adoption strategies, engine calibration processes, and sustainable transportation initiatives to help minimize fossil-fuel dependency and greenhouse-gas emissions.

The proposed SF-CRITIC-RAPS framework is primarily intended as an **offline decision-support tool** for selecting the most suitable biodiesel blend based on engine performance, combustion characteristics, fuel economy, and emission criteria. Since conventional diesel engines operate with a pre-selected fuel, real-time switching between biodiesel blends is generally not feasible without specialized multi-fuel storage and blending hardware. Nevertheless, the framework can be integrated with modern **Engine Control Units (ECUs)**, **IoT-enabled engine monitoring systems**, and **digital twin platforms** to support real-time engine calibration, combustion optimization, emission monitoring, and adaptive control. In such applications, real-time sensor measurements (e.g., engine speed, load, in-cylinder pressure, exhaust temperature, and emissions) can be continuously analyzed to adjust injection timing, injection pressure, exhaust gas recirculation (EGR), and other control parameters while operating with the selected fuel blend. This integration enhances engine efficiency and emission performance without requiring real-time fuel replacement.

5.5. Limitations

Although the findings of this study are encouraging, there are some drawbacks. The

analysis was based on one experimental dataset of the CTBD and might not be representative of the results of the analysis of other experimental datasets for other engine types and operating conditions. Second, even though the uncertainty modeling ability of spherical fuzzy sets allows for the inclusion of some form of judgments, the expert assessments themselves can create judgmental bias. Third, the proposed model is centered on performance, combustion, and emission parameters and does not include any economic parameters like fuel production cost, lifecycle cost, maintenance cost, etc. Lastly, the study is based on steady-state operating conditions and does not include the transient behavior of the engine found in actual driving situations.

5.6. Future Research Directions

The proposed framework can be expanded in future research to include larger, different biomass sources, hydrogen blended fuels, and advanced combustion technologies. Incorporating machine learning and deep learning approaches with spherical fuzzy MCDM models might be a promising approach for further improving prediction and optimization capability. Moreover, further investigations could be conducted on hybrid combinations like SF-CRITIC-MARCOS, SF-CRITIC-TOPSIS, and SF-CRITIC-CoCoSo to examine the stability of the ranking results obtained from the different frameworks of decision making. Other considerations, such as economic, environmental, and life cycle assessment (LCA), should be integrated to aid in a holistic assessment of sustainability. Lastly, an intelligent engine management system with Internet of Things (IoT) sensors could be designed for real-time optimization of blends and adjustment of engine control functions depending on the operating conditions.

6. CONCLUSION

The present study introduced a novel Spherical Fuzzy CRITIC-RAPS framework for the optimization of combustion engine performance by using the CTBD for the combustion engine performance, combustion, and emission dataset. The spherical fuzzy theory and the objective CRITIC weighting and RAPS ranking methods were well combined to overcome the uncertainty and conflicting evaluation criteria. Based on the results, CTBD30 presented the best fuel performance, combustion efficiency, fuel economy, and reduction in emissions, indicating the optimal fuel blend. The approach was also found to be robust and reliable through comparative and sensitivity analysis. The study provides a practical decision support model for selecting fuels for sustainable cars in automotive industry, and emphasizes the importance of using advanced fuzzy MCDM models for multi-objective engine optimization. Future work will involve combining economic and lifecycle assessment

criteria, larger datasets, and intelligent optimization techniques to further improve the application in real-world scenarios.

Declarations

Data and material availability: Upon reasonable request, the corresponding author will provide any data created or analyzed during this investigation.

Competing interests: Regarding the research, writing, and/or publication of this article, the author or authors have disclosed no possible conflicts of interest.

Ethical approval & Informed Consent: This study did not include human subjects. The data used in this study is completely fictitious and was created to illustrate the suggested technique. Therefore, ethical approval and informed consent were not necessary.

Authors contributions: Zhenshan Fan wrote the abstract, extension of the CRITIC method, and RAPS method in the SFS environment, and a case study. Yan Zhang built the introduction, Background, and sensitivity analysis and comparative analysis and conclusion of the article and supervised the manuscript. Both authors approved the manuscript.

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Informed consent: No research involving human subjects is included in this article.

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