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DURABILITY ASSESSMENT OF RAILWAY CAR AXLES TAKING INTO ACCOUNT THE CHARACTER OF THE STRESS SPECTRUM

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Abstract

In many countries, railway cars with axles that have operated for over 50 years are still in use, which significantly exceeds their normative service life. This can be explained by the fact that design calculations are based on overestimated loads, as evidenced by published information on diagnostics of railway axles, which is performed under real operating conditions. In this work proposes a method for calculating the durability of axles of railway cars under real operating conditions, which is based on fatigue tests of samples made of axle material, the statistical theory of similarity of fatigue failure, analysis of the stress state and known models of fatigue curves. The calculations performed showed that, under real operating conditions, the durability of the axles exceeds $N=10^{10}$ cycles, which indicates their operation in the field of gigacycle fatigue. These values N correspond to a service life of over 30 years. Recommendations are given for assessing the durability of railway axles, taking into account actual operating conditions.

Keywords: fatigue curve models, similarity theory of fatigue failure, stress spectrum of axles

1. INTRODUCTION

The durability of the axles is ensured in the following stages:

– design (choice of material, level of rationality);
– manufacturing (material quality, thermal and mechanical treatment, assembly of wheelsets);
– operation (nature and levels of static and dynamic loads, etc.).

If the requirements of the manufacturing technology are met and the quality of the material is ensured, the main cause of destruction (if emergency situations are not taken into account) is the degradation of the mechanical properties of the axle material during long-term operation. This phenomenon is mainly due to the accumulation of fatigue damage due to high-cycle loading, which leads to the appearance of microcracks that are concentrated in zones of maximum stress, that is, near stress concentrators.

High requirements for the reliability of railway axles determine the appointment of inflated safety margins, which lead to excess durability, as evidenced by published information on the results of axles operation [1, 2].

Calculation of the durability of railway axles must be carried out taking into account the magnitude of their loads under operating conditions and the characteristics of the accumulation of fatigue damage. The last factor is characterized by the specificity of fatigue curve models for the region of high durability, which are considered in this work.

2. PROBLEM ANALYSIS AND TASK STATEMENT

One of the main tasks of calculating railway axles is a determine of their durability, which should take into account, along with the mechanical characteristics of the material, their loading conditions. With a wide variety of these conditions, it is worth noting their common feature, established by numerous observations – they relatively rarely experience the action of maximum permissible loads [3-5].

As an example, is demonstrated the experimentally established stress spectrum of the axles of railway cars, presented in [6] in the form of a seven-stage stresses block, Tab. 1. It was obtained as a result of full-scale tests using the strain gauge

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method and, therefore, takes into account the characteristics of real loading, including the dynamic components.

Table 1. Block of stresses spectrum of railway axles [6]

σ_i, MPa	159	150	130	105	85	62	50
$n_{bi}\cdot 10^{-3}$, cycles	0.03	0.1	1.3	6	26	140	550

The total quantity of cycles in this block is $\sum n_{bi}$=723430 cycles, of which the duration actions of the greatest stresses (σ_1=159 MPa and σ_2=150 MPa) is only 0.018 %. Presented in Tab. 1 block of the stress's spectrum according to the classification of typical modes corresponds to especially light [7]. This explains the increased reliability of railway axles and their long service life.

In the experiment, using strain gauges that are installed in the middle cylindrical region of the axis, normal stresses are determined σ_x. Outside this region, in the area between the wagon wheel and the journal, tangential stresses arise during braking τ_{xt}. It is shown below that the magnitude of these stresses is insignificant, so they will not be taken into account in the calculation.

Existing axle design methods are based on ensuring a safety margin and do not explicitly define their service life, which does not allow establishing the time for safe operation of the axle and the frequency of their inspection [8].

The purpose of this work is a computational and experimental assessment of the durability of railway axles under real operating conditions. The calculation method uses:
– results of fatigue tests of samples made of axle material;
– determination of axle endurance limits in dangerous sections;
– axle fatigue curves parameters;
– block of the stress's spectrum, which act during operation of railway axles.

3. OBJECT OF RESEARCH

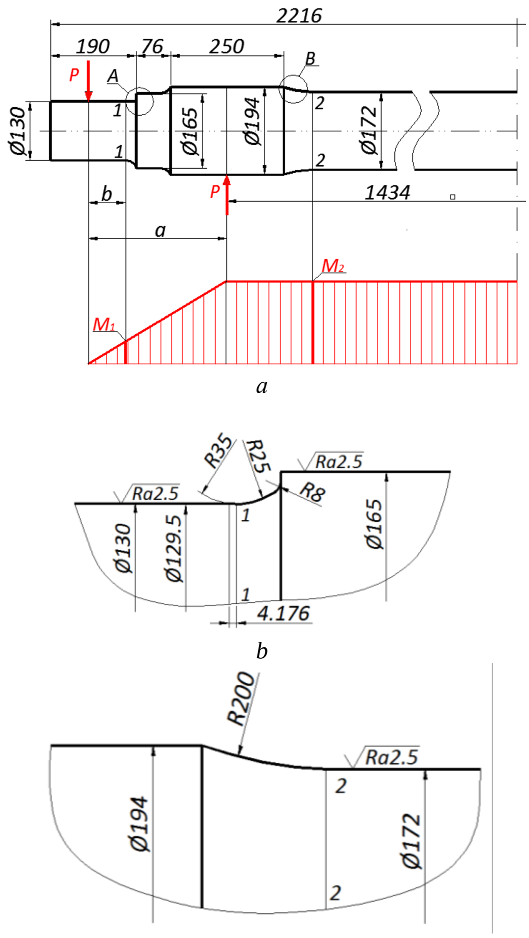
The implementation of the methodology for computational and experimental assessment of durability is carried out in relation to the standard axle of railway cars [7, 9, 10]. Axles for rail vehicles are usually made of medium-carbon steels [2, 9]. In this work, calculations were performed for an axle made of steel 45. The main components of this steel: C – 0.45 %, Si – 0.19 %, Mn – 0.5 %, Ni – 0.2 %, Cr – 0.2 %, Cu – 0.2 %, S – 0.03 %, P – 0.03 % (the rest is Fe). Mechanical characteristics: ultimate strength σ_u=640 MPa, yield stress σ_y=350 MPa, elongation at break δ=19 %, hardness (177...185) HB.

The maximum calculated static load of the wheel pair on the rails for a passenger car is *P*=176.6 kN, for a freight car – *P*=230.5 kN, therefore, the durability calculation is performed for the axle of a freight cars. Due to the symmetry of the structure and the loading relative to the middle, half of the axle is shown here, Fig. 1.

From the analysis of the design of the axle and the diagram of bending moments (Fig. 1) it follows that dangerous are:
a) section 1-1 in the area of fillet *A*, which has a smallest diameter of 129.5 mm and a rounding radius of 35 mm;
b) section 2-2 in the area of fillet *B*, where operates the greatest bending moment M_2 and a rounding radius is 200 mm.

To analyze the nature of the distribution of stresses arising in the axle under the action of a rated load, its calculation was carried out using the finite element method, Fig. 2.

It was established that in dangerous sections 1-1 and 2-2 the values of the maximum bending stresses are respectively equal to 148.8 MPa and 163 MPa, Fig. 2. Therefore, axle durability calculations must be performed for these sections.



a

b

c

Fig. 1. Non-driven railway axle:
a – design scheme and diagram of bending moments; *b*, *c* – geometric characteristics of fillets *A* and *B*, respectively

The wagon is braked by the braking system. In the case of a two-shoe brake, the braking torque is created by pressing the shoes against the wheel. Its magnitude $T_b = N_b f D_w$, where $N_b = 3800$ N – the force of pressing the pads onto a wheel with a diameter of $D_w = 840$ mm, $f = 0.2$ – coefficient of friction for a steel-cast iron pair [11].

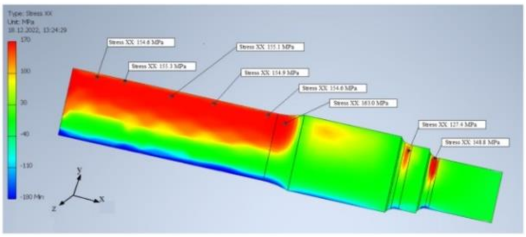


Fig. 2. Stress distribution along the axle length

The calculations give the following values: $T_b = 3800 \cdot 0.2 \cdot 840 = 638400$ Nmm, polar moment of resistance in dangerous section 1-1 (Fig. 1, b) $W_{p1} = \pi d_1^3/16 = 3{,}14 \cdot 129{,}5^3/16 = 426400$ mm^3. Maximum shear stresses in this dangerous section $\tau_{xt} = T_b/W_{p1} = 1.5$ MPa. Stresses from bending of the axis in section 1-1, obtained in the work $\sigma_x = 148.8$ MPa and equivalent stresses $\sigma_{eq} = \sqrt{\sigma_x^2 + 4\tau_{xt}^2} = 148.83$ MPa. Consequently, the magnitude of the stresses τ_{xt} arising in the dangerous section 1-1 is insignificant compared to the stresses from bending σ_x which act in this section. Therefore, the tangential stresses are not taken into account in the calculation.

4. DETERMINATION OF AXLE ENDURANCE LIMITS IN DANGEROUS SECTIONS

In this method, to assess the durability of a railway axle, its endurance limit should be established. Fatigue testing of the axle itself is difficult due to the complexity of conducting it. The assessment of endurance limits in dangerous sections 1-1 and 2-2 is carried out using the statistical theory similarity of fatigue failure based on the results of testing laboratory samples [12]. In the indicated sections of the axle, the endurance limits σ_{-1D1} and σ_{-1D2} under a symmetrical loading cycle are determined by the formulas [7, 10, 12]

$$\sigma_{-1D1} = \frac{\bar{\sigma}_{-1} K_{V1} K_{A1}}{\frac{2K_{t1}}{1+\theta_1^{-\nu_\sigma}} + \frac{1}{K_{F1}} - 1}$$

$$\sigma_{-1D2} = \frac{\bar{\sigma}_{-1} K_{V2} K_{A2}}{\frac{2K_{t2}}{1+\theta_2^{-\nu_\sigma}} + \frac{1}{K_{F2}} - 1}, \qquad (1)$$

where $\bar{\sigma}_{-1}$ – the average value of the endurance limit of the axle material (smooth laboratory sample with diameter $d_0 = 7.5$mm) under a symmetrical loading cycle; K_{V1} and K_{V2}– coefficients of influence of surface hardening; K_{A1} and K_{A2}– anisotropy coefficients; K_{t1} and K_{t2} – theoretical stress concentration coefficients; θ_1 and θ_2– relative criteria for the similarity of fatigue failure; ν_σ – coefficient of sensitivity of the metal to stress concentration and to scale factor; K_{F1} and K_{F2} – surface roughness coefficients.

To reliably determine the coefficient, it is recommended to carry out fatigue tests of several batches of samples made of axle material with different geometric characteristics [12]. For this purpose, fatigue tests were performed at bending with rotation of the samples:
– corsetry (types I–III);
– V-notched (types IV–VII).

The samples were made of steel 45 (heat treatment – normalization), Fig. 3, Tab. 2.

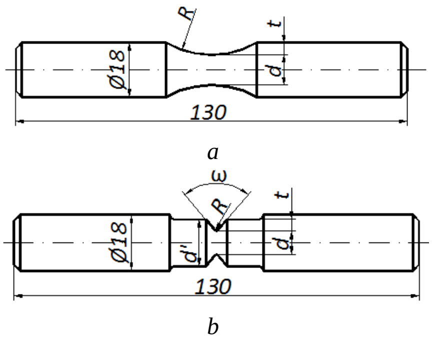


Fig. 3. Design of laboratory samples for fatigue testing: *a* – corsetry; *b* – V-notched

The endurance limits for the tested samples were established by the "up-down" method based on the results of testing batches of 20 samples of each type up to the base $N_B = 10^7$ of loading cycles.

The ν_σ coefficient is determined by the similarity equation for fatigue failure at a probability of 50 %. This equation in a logarithmic coordinate system has a linear form [12]

$$log(\xi - 1) = -\nu_\sigma \, log\, \theta. \qquad (2)$$

Table 2. Geometric characteristics of samples

Sample type	Dimensions, mm d'	d	t	R	Angle ω
I	18	7.5	5.25	25	–
II		10	4	30	–
III		15	1.5	75	–
IV	15	7.5	3.75	0.25	42°42′
V	18	15	1.5	0.3	55°33′
VI		7.5	5.25	2	86°52′
VII		10	4	4	–

In formula (2), the relative maximum stress

$$\xi = \frac{\sigma_{max}}{u \frac{2\sigma_{-1}\cdot K_t}{\bar{\sigma}_{-1}}}, \qquad (3)$$

where is the maximum value of stresses in the zone of their concentration $\sigma - 1_{t_{max}}$; minimum damaging stress, which is assumed to be equal to half the endurance limit of a smooth laboratory sample [10, 12]

$$u = 0.5\bar{\sigma}_{-1}\,. \qquad (4)$$

Theoretical stress concentration coefficients are calculated using the formula for a cylindrical sample with hyperbolic annular cuts (grooves) during bending, given in [10]

$$K_t = 1 + \frac{(K_{tf}-1)(K_{tt}-1)}{\sqrt{(K_{tf}-1)^2 + (K_{tt}-1)^2}}, \qquad (5)$$

where the theoretical stress concentration coefficients for shallow and deep cuts, respectively, were calculated using the formulas

$$K_{tf} = 1 + 2\sqrt{t/\rho},$$

$$K_{tt} = \frac{3}{4}\frac{\left(1+\sqrt{\frac{a}{\rho}+1}\right)\left(3\frac{a}{\rho}+4.3-0.4\sqrt{\frac{a}{\rho}+1}\right)}{3\left(\frac{a}{\rho}+1\right)+2.2\sqrt{\frac{a}{\rho}+1}+1.3/\left(1+\sqrt{\frac{a}{\rho}+1}\right)}., \quad (6)$$

In formula (6) $a = d/2$ is the radius of the dangerous section; $\rho = R$ – for corsetry samples; $\rho = R/1.05$ is the reduced radius of curvature for V-notched samples when replacing the hyperbolic profile with an equivalent straightened profile [10].

The relative similarity criterion for fatigue failure is a ratio [10, 12]

$$\theta = \frac{L/\bar{G}}{L_0/\bar{G}_0}, \quad (7)$$

where respectively $L=\pi d$ and $L_0=\pi d_0$ are the perimeters of dangerous sections of samples of types I–VII (Tab. 2) and a smooth laboratory sample with a diameter of d_0=7.5 mm; the relative gradients of the first principal stress in the zone of the sections under consideration are respectively equal to $\bar{G}_0 = 2/d_0 = 2/7.5 = 0.2667$ mm^{-1} and $\bar{G}$ - for a smooth sample and samples of types I–VII,

$$\bar{G} = \frac{2.3(1+\phi\phi)}{R} + \frac{2}{d}. \quad (8)$$

In formula (8) the parameter $\phi\phi = \frac{1}{4\sqrt{t/R}+2}$ at $D/d < 1.5$ and $\phi\phi = 0$, at $D/d \geq 1.5$.

The results of calculations using the above formulas are presented in table Tab. 3.

Table 3. Parameters of equations (2), (3) for tested samples

Sample type	θ	σ_{-1}, MPa	K_t	σ_{max}, MPa
I	0.7692	325	1.043	338.98
II	1.3333	300	1.048	314.40
III	3.1300	293	1.029	301.50
IV	0.0308	150	3.123	468.45
V	0.0687	120	3.534	424.08
VI	0.2025	245	1.471	360.40
VII	0.4904	260	1.331	346.06

The endurance limit of a smooth laboratory sample is determined according to the method described in [12]. To this is set the approximate value $\bar{\sigma}'_{-1} = 325$ MPa (from the tested samples, the one with $d_0 = 7.5$mm and the shape of the working part that is closest to smooth is selected – type I). Then the corresponding pairs of $log(\sigma_{max} - u') = log(\sigma_{max} - 0.5\bar{\sigma}'_{-1}) = log(\sigma_{max} - 162.5)$ and $log\,\theta$ values are calculated for various types of samples, Tab. 4.

Based on the results of statistical processing of these pairs of values using the least squares method, a linear equation was obtained. (9)

To a smooth laboratory sample is corresponds the value $log\,\theta = 0$. From formula (9) with this value of the argument is obtained. Hence, the endurance limit $\bar{\sigma}_{-1}$ of a smooth laboratory sample (which has theoretical stress concentration coefficient $K_t = 1$), $\bar{\sigma}_{-1} = \sigma 2.2135\text{MPa}_{max}$.

Table 4. Data for calculating the coefficient ν_σ

Sample type	$\log\theta$	$\log(\sigma_{max}-162.5)$	$\log(\xi-1)$
I	-0.1139	2.0732	0.0307
II	0.1249	1.9229	–0.0348
III	0.4955	1.8440	–0.0736
IV	–1.5119	2.8651	0.2707
V	–1.1629	2.5938	0.2024
VI	–0.6935	2.2043	0.0807
VII	–0,3094	2.1166	0.0479

The minimum damaging stress is calculated using formula (4) $u = 0.5\bar{\sigma}_{-1} = 163.5$ MPa. The relative maximum stresses ξ, found by formula (3) for the tested types of samples and the corresponding values of $log(\xi - 1)$ are given in Tab. 4.

The coefficient ν_σ is the tangent of the angle of inclination between the abscissa axis x $= log\,\theta$ and line (2). This coefficient is determined by processing pairs of $x_i = log\,\theta_i$ and $y_i = log(\xi_i - 1)$ values (Tab. 4) using the least squares method

$$\nu_\sigma = -\frac{\sum_{i=1}^{7} x_i \cdot y_i}{\sum_{i=1}^{7} x_i^2} = 0.1712. \quad (10)$$

The graph of equation (2) $log(\xi - 1) = -0.1712\,log\,\theta$ at the found value of the coefficient ν_σ (10) is shown in Fig. 4.

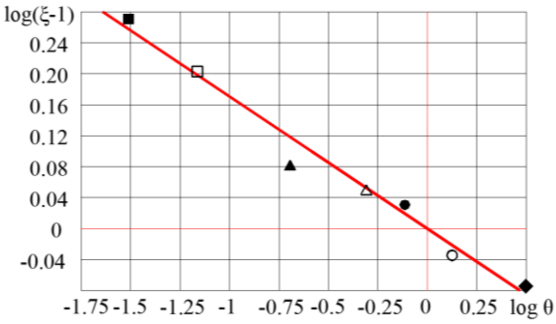


Fig. 4. Graphical representation of equation (2) for samples:
● – type I, ○ – type II, ◆ – type III, ■ – type IV, □ – type V, ▲ – type VI, △ – type VII

The remaining parameters of equation (1) for dangerous sections of the axle 1-1 and 2-2 were found according to the recommendations [12]. In the following, all parameters for sections 1-1 and 2-2 are assigned indices 1 and 2, respectively. Coefficient of influence of surface hardening $K_{V1} = K_{V2} = 1$ (no hardening); anisotropy coefficients $K_{A1} = K_{A2} = 0.86$ (at σ_u=640 MPa>600 MPa); surface roughness coefficients

$$K_{F1} = K_{F2} = 1 - 0.22\left(log\frac{\sigma_u}{20} - 1\right) \cdot log\,R_z =$$
$$= 1 - 0.22\left(log\frac{640}{20} - 1\right) \cdot log\,1\,0 = 0.889,$$

where is the surface roughness of fillets *A* and *B* of the considered axle $R_a 2.5 = R_z 10$, Fig. 1, *b* and *c*.

For further calculations of the axle, the following size designations are accepted:
– fillet *A*: $d_1 = 129.5$ mm, $D_1 = 165$ mm, $R_1 = 35$ mm;
– fillet *B*: $d_2 = 172$ mm, $D_2 = 194$ mm, $R_2 = 200$ mm.

Theoretical stress concentration coefficients during bending with rotation for fillets *A* and *B*, respectively, are calculated using the formulas [12]

$$K_{t1} = 1 + \frac{1}{\sqrt{\frac{0.62}{\tau_1} + 5.8\frac{(1+\alpha_1)^2}{\alpha_1^3} + \frac{0.2}{\tau_1^3}\cdot\frac{\alpha_1}{\alpha_1+\tau_1}}} = 1.318$$

where $\alpha_1 = d_1/(2R_1) = 129.5/(2\cdot 35) = 1.85$ mm ;$\tau_1 = \frac{(D_1-d_1)}{2R_1} = \frac{165-129.5}{2\cdot 35} = 0.507$ mm and

$$K_{t2} = 1 + \frac{1}{\sqrt{\frac{0.62}{\tau_2} + 5.8\frac{(1+\alpha_2)^2}{\alpha_2^3} + \frac{0.2}{\tau_2^3}\cdot\frac{\alpha_2}{\alpha_2+\tau_2}}} = 1.029$$

where $\alpha_2 = d_2/(2R_2) = 172/(2\cdot 200) = 0.43$ mm ;

$$\tau_2 = \frac{(D_2-d_2)}{2R_2} = \frac{194-172}{2\cdot 200} = 0.055 \text{ mm}.$$

Relative criteria for the similarity of fatigue failure are determined by (7) $\theta_1 = \frac{L_{D1}/\bar{G}_{D1}}{L_0/\bar{G}_0}$ and $\theta_2 = \frac{L_{D2}/\bar{G}_{D2}}{L_0/\bar{G}_0}$, where, respectively, L_{D1} and L_{D2} – the perimeters of dangerous sections 1 and 2, $\bar{G}_{D1}$ and $\bar{G}_{D2}$ – the relative gradients of the first principal stress in the zone of the sections under consideration. By formulas from [12], are found $L_{D1} = \pi d_1 = \pi \cdot 129.5 = 406.84$ mm, $L_{D2} = \pi d_2 = \pi \cdot 172 = 540.35$ mm. Relative gradient for section 1-1

$$\bar{G}_{D1} = \frac{2.3(1+\phi_1)}{R_1} + \frac{2}{d_1} = \frac{2.3(1+0.206)}{35} + \frac{2}{129.5} = = 0.09471 \text{ mm}^{-1},$$

where, under the condition $D_1/d_1 = 165/129.5 = 1.27 < 1.5$, the parameter is defined

$$\phi\phi\phi_1 = \frac{1}{4\sqrt{t_1/R_1}+2} = \frac{1}{4\sqrt{17.75/35}+2} = 0.206.$$

Relative gradient for section 2-2

$$\bar{G}_{D2} = \frac{2.3(1+\phi_2)}{R_2} + \frac{2}{d_2} = \frac{2.3(1+0.340)}{200} + \frac{2}{172} = 0.02704 \text{ mm}^{-1}..$$

Relative gradient for section 2-2 $D_2/d_2 = 194/172 = 1.13 < 1.5$ the parameter is defined

$$\phi\phi\phi_2 = \frac{1}{4\sqrt{t_2/R_2}+2} = \frac{1}{4\sqrt{11/200}+2} = 0.340.$$

Relative criteria for similarity of fatigue failure for dangerous sections

$$\theta_1 = \frac{L_{D1}/\bar{G}_{D1}}{L_0/\bar{G}_0} = \frac{406.84/0.09471}{23.56/0.2667} = 48.615,\ \theta_2 = \frac{L_{D2}/\bar{G}_{D2}}{L_0/\bar{G}_0} = \frac{540.35/0.02704}{23.56/0.2667} = 226.15$$

By formula (1), the endurance limits for these sections are calculated

$$\sigma_{-1D1} = \frac{327\cdot 1\cdot 1}{\frac{2\cdot 1.318}{1+48.615^{-0.1718}} + \frac{1}{0.889} - 1} = 150.7 \text{ MPa},$$

$$\sigma_{-1D2} = \frac{327\cdot 1\cdot 1}{\frac{2\cdot 1.029}{1+226.15^{-0.1718}} + \frac{1}{0.889} - 1} = 175.1 \text{ MPa}.$$

5. ASSESSMENT OF THE DURABILITY OF A RAILWAY AXLE

To determine durability of axle must be known:
– fatigue curve model with its parameters;
– endurance limits in dangerous sections;
– spectrum of acting stresses.

The stress spectrum is determined during operational tests of the axles over a sufficiently long period of time and is presented in the form of a step diagram (see Tab. 1). The measurement results are obtained for sections of the axle where there are no stress concentrators, in this case – on the cylindrical surface of the axle, where the bending moment *M*=const, Fig. 1. This statement is confirmed by calculating the axle using the finite element method, Fig. 2, where thenstress $\sigma_c = 155$ MPa acts within this cylindrical surface. This calculated stress correlates satisfactorily with the experimentally determined maximum value of $\sigma_e = 159$ MPa (Tab. 1), since the accuracy of the calculation performed by the finite element method was set to 3 %.

In dangerous sections of axes 1-1 and 2-2, the stress values differ from those measured in the experiment, and therefore, to determine the durability, the stress values were recalculated in relation to both sections under consideration, where the stresses are 148.8 MPa and 163 MPa, respectively, Fig. 2. Obtained as a result of recalculation the values stresses of block (Tab. 1) are presented in Tab. 5 and 6.

Table 5. Stresses spectrum block in section 1-1

σ_i, MPa	148.8	140.4	121.7	98.3	79.5	58	46.8
$n_{bi}\cdot 10^{-3}$, cycles	0.03	0.1	1.3	6	26	140	550

Table 6. Stresses spectrum block in section 2-2

σ_i, MPa	163	153.8	133.3	107.6	87.1	63.6	51.3
$n_{bi}\cdot 10^{-3}$, cycles	0.03	0.1	1.3	6	26	140	550

For dangerous areas of the axle, the endurance limits are established in the previous section.

Under cyclically changing stresses, damage accumulates in the material of the axles, which manifests itself in changes in mechanical properties due to the evolution of defects that are formed at the stage of creating parts. Numerous observations indicate degradation of cyclic strength over time of operation. The degradation of material properties under the influence of long-term operation of parts is taken into account by adjusting fatigue curve models. The work discusses three models of fatigue curves that can be used to calculate the durability of railway axles which a long time are in operation.

Model 1. In traditional calculations, to describe the state of parts under cyclic loading, the Basquin fatigue curve ABB_1 is used (Fig. 5), the mathematical model of which is presented in the form [13, 14]

$$\begin{cases} \sigma_D^m N = \sigma_{-1D}^m N_G = 10^C, \text{ if } \sigma_D \geq \sigma_{-1D} \\ \sigma_D = \sigma_{-1D}, \text{ if } N \geq N_G \end{cases} \quad (11)$$

where σ_D and N – the current stress value and the corresponding durability before failure; m and C – parameters of equation (11); σ_{-1D} – the endurance limit of the axle under a symmetrical stress cycle, N_G – the abscissa of the break point of the fatigue curve plotted in double logarithmic coordinates.

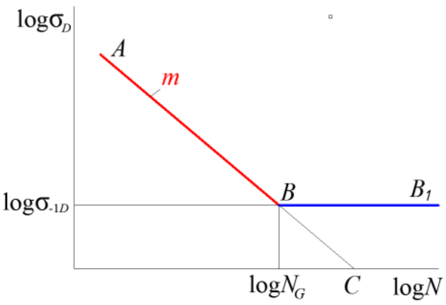


Fig. 5. Fatigue curve ABB_1 according to (11)

The parameters of equation (11) are determined for each dangerous section of the axle individually according to the method proposed in [7]. For this purpose the stresses σ_i, at which fatigue tests of smooth laboratory samples were carried out are reduced by a fixed value $\Delta = \sigma_{-1} - \sigma_{-1D}$ for each value of durability before failure N_i. All stress values at which type I samples were tested were subjected to this adjustment, Tab. 3. For the considered dangerous sections of the axle the values of the indicated differences

$$\Delta_1 = \sigma_{-1} - \sigma_{-1D1} = 174.3 \text{ MPa}, \Delta_2 = \sigma_{-1} - \sigma_{-1D2} = 149.9 \text{ MPa}.$$

Logarithms of stress values $\sigma_{D1,i} = \sigma_i - \Delta_1$, $\sigma_{D2,i} = \sigma_i - \Delta_2$ and the corresponding values of logarithms of durability were processed using the least squares method. For the sections under consideration, the parameters of (11) were obtained:

$$m_1 = 11.53, N_{G1} = 1.488 \cdot 10^6 \text{ cycles}, C_1 = 31.29; m_2 = 13.01, C_2 = 35.36, N_{G2} = 1.453 \cdot 10^6 \text{ cycles},$$

at which high correlation coefficients are ensured, respectively $r_1 = -0.955$ and $r_2 = -0.956$.

Assessment of the durability of parts under stepwise cyclic loading is carried out using the linear hypothesis of damage summation

$$\sum_{i=1}^{p}(n_i/N_i) = a, \quad (12)$$

where n_i – the operating time in cycles at the i-th stage (by stage we mean the stress level); N_i – the durability before failure according to the fatigue curve at the same stage, p – the quantity of loading stages a – the sum of accumulated damage.

Formula (12), taking into account equation (13), is transformed to the form

$$\sum_{i=1}^{p}(\sigma_{Di}^m n_i/10^C) = a \quad (13)$$

Stress levels $\sigma_{Di} \leq \sigma_{-1Di}$ should not be taken into account when summing according to (13), which follows from condition (11). In this case, the quantity of terms in (13) may be less than the specified quantity of stages p.

The procedure for calculating durability is demonstrated using the stress spectra of the axles of railway cars given in Tab. 5 and 6. The total quantity of cycles in block $n_b = \sum_{i=1}^{7} n_{bi} = 723430$ cycles (this quantity of cycles corresponds to the mileage $l_0 = \pi D_w n_b \cdot 10^{-6} = 1909$ km with a railway car wheel diameter D_w=840 mm). The destruction of the axle occurs as a result of the influence of a certain quantities of blocks M, at which the sum of accumulated damage reaches the value a=1, that is, in accordance with formulas (12), (13)

$$\sum_{i=1}^{p}(M \cdot n_{bi}/N_i) = M \sum_{i=1}^{p}(\sigma_{Di}^m \cdot n_{bi}/10^C) = 1. \quad (14)$$

In the calculation determines the quantity blocks of stresses M before the expected destruction of the axle

$$M = 10^C / \sum_{i=1}^{p} \sigma_{Di}^m \cdot n_{bi}. \quad (15)$$

The durability of an axle in dangerous sections is estimated by the quantity of cycles to failure

$$N_\Sigma = M \sum_{i=1}^{p} n_{bi}. \quad (16)$$

or by mileage (km)

$$L_s = M \cdot l_0. \quad (17)$$

In formulas (15), (16), the quantity of taken into account stages p of the loading block (Tab. 5 and 6) depends on the magnitude of the axle endurance limits in dangerous sections 1-1, 2-2, see Fig. 1.

The results of calculations using formulas (15)-(17) are presented in Tab. 7.

Table 7. Parameters of fatigue curves and calculation results for the Basquin model

Characteristics	Section 1-1	Section 2-2
σ_{-1D}, MPa	150.7	175.1
m	11.53	13.01
C	31.29	35.36
p	0	0
M, by (15)	infinity	infinity
N_Σ by (16), cycles	infinity	infinity
L_s by (17), km	infinity	infinity

Model 2. Model 2 is based on the Haibach hypothesis [15, 16], according to which the fatigue curve has a kink at point B, Fig. 6.

Here it is believed that the entire spectrum of stresses is involved in the accumulation of damage, and the fatigue curve at $\sigma_{Di} \leq \sigma_{-1Di}$ has increased parameters, $m_H > m$ and $C_H > C$.

The mathematical notation of the fatigue curve for model 2 has the form

$$\begin{cases} \sigma_D^m N = \sigma_{-1D}^m N_G = 10^C, \text{ if } \sigma_D \geq \sigma_{-1D} \\ \sigma_D^{m_H} N = \sigma_{-1D}^{m_H} N_G = 10^{C_H}, \text{ if } \sigma_D \leq \sigma_{-1D} \end{cases}, \quad (18)$$

where m, m_H, C, C_H – parameters.

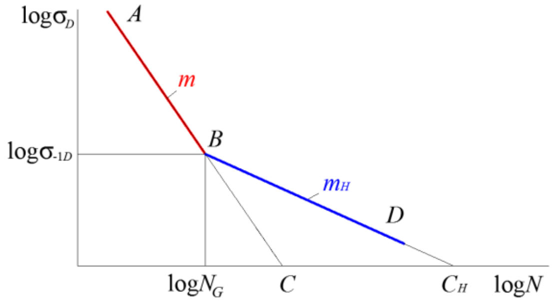


Fig. 6. Haibach fatigue curve in logarithmic coordinates

The calculation procedure according to (18) was performed for the accepted load spectrum, Tab. 5 and 6. The parameters m and C of the left branch of the fatigue curve were calculated using the same method as when considering model 1. Parameter m_H of the right branch of the fatigue curve is determined by the formula proposed in [15, 16]

$$m_H = 2m - 1. \quad (19)$$

The parameter C_H is calculated by solving both equations of system (18), written for their common point B

$$C_H = C + (m_H - m) \log \sigma_{-1D}. \quad (20)$$

The sums of accumulated damage a_1 and a_2 were calculated separately using the left and right branches of the fatigue curve (Fig. 6) using the formulas

$$\begin{cases} a_1 = M \sum_{i=1}^{q} \frac{n_{bi}}{N_i} = \frac{M}{10^C} \sum_{i=1}^{q} \sigma_{Di}^m n_{bi}, \text{ if } \sigma_D \geq \sigma_{-1D} \\ a_2 = M \sum_{i=q+1}^{p} \frac{n_{bi}}{N_i} = \frac{M}{10^C} \sum_{i=q+1}^{p} \sigma_{Di}^{m_H} n_{bi}, \text{ if } \sigma_D < \sigma_{-1D} \end{cases}, \quad (21)$$

where q – the quantity of stages with stresses $\sigma \geq \sigma_{-1Di}$.

The limit state is determined by the condition a_1+a_2=1. In this case from (21) the quantity blocks of stresses is calculated

$$M = \left(10^{-C} \cdot \sum_{i=1}^{q} \sigma_{Di}^m n_{bi} + 10^{-C_H} \cdot \sum_{i=q+1}^{p} \sigma_{Di}^{m_H} n_{bi}\right)^{-1}. \quad (22)$$

The durability of the axle is determined by formulas (16), (17), (22). The calculation results for model 2 are presented in Tab. 8.

Table 8. Parameters of fatigue curves and calculation results for model 2

Characteristics	Section 1-1	Section 2-2
σ_{-1D}, MPa	150.7	175.1
m	11.53	13.01
C	31.29	35.36
m_H	22.06	25.02
C_H	54.23	62.30
M, by (22)	26896	144634
N_Σ, cycles, by (16)	$1.946 \cdot 10^{10}$	$1.046 \cdot 10^{11}$
L_s, km, by (17)	$5.134 \cdot 10^7$	$2.761 \cdot 10^8$

Model 3. Analysis of studies of steel parts that failed from long-term cyclic stresses shows that the fatigue curve has a horizontal section at level σ_{-1D} in the durability range of 10^6÷10^9 cycles. Based on the studies conducted, a four-link model of the fatigue curve was proposed (it is called duplex) [17-20], Fig. 7. In this work, the following assumptions are made to describe this model:

– the horizontal section on the fatigue curve has a length from N_G to 10^8 cycles;

– the height of the stage is $0.5\sigma_{-1D}$;

– the parameter m of the inclined branch of the fatigue curve remains unchanged.

Based on these assumptions, a mathematical formulation of model 3 is obtained

$$\begin{cases} \sigma_D^m N = \sigma_{-1D}^m N_G = 10^C, \text{ if } \sigma_D \geq \sigma_{-1D} \\ \sigma_D = \sigma_{-1D}, \text{ if } N_G \leq N \leq 10^8 \\ \sigma_D^m N = (0.5\sigma_{-1D})^m N_G' = 10^{C_d}, \text{ if } 0.5\sigma_{-1D} \leq \sigma_D \leq \sigma_{-1D} \\ \sigma_D = 0.5\sigma_{-1D}, \text{ if } N \geq N_G' \end{cases} \quad (23)$$

Dependences (23) in logarithmic coordinates are described by the broken line ABB_1FG.

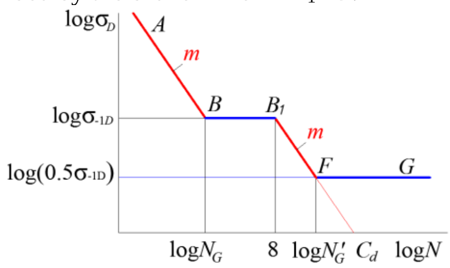


Fig. 7. Duplex model of the fatigue curve in the form (23) in logarithmic coordinates

The durability calculation according to model 3 was also performed for a given range of loads, Tab. 7 and 8. Determination of parameters m and C for section AB of the fatigue curve is performed in the same way as for models 1 and 2. For zone B_1F, the parameter m remains the same, and the value of the parameter C_d is determined from the third equation of the system (23)

$$C_d = 8 + m \log \sigma_{-1D}. \quad (24)$$

The sums of accumulated damage a_1 and a_2 were calculated separately for the inclined sections of the fatigue curve (Fig. 7) using the formulas

$$\begin{cases} a_1 = M \sum_{i=1}^{q} \frac{n_{bi}}{N_i} = \frac{M}{10^C} \sum_{i=1}^{q} \sigma_{Di}^m n_{bi}, \text{ if } \sigma_D \geq \sigma_{-1D} \\ a_2 = M \sum_{i=q+1}^{p} \frac{n_{bi}}{N_i} = \frac{M}{10^{C_d}} \sum_{i=q+1}^{p} \sigma_{Di}^m n_{bi}, \text{ if } \frac{\sigma_{-1D}}{2} \leq \sigma_D < \sigma_{-1D} \end{cases}$$

where q – the quantity of loading stages with stresses $\sigma \geq \sigma_{-1Di}$; p – the quantity of loading stages with stresses $0.5\sigma_{-1D} \leq \sigma_D \leq \sigma_{-1D}$.

In accordance with the condition a_1+a_2=1, a formula was obtained for determining the durability of the axle through the quantity blocks of loading

$$M = \left(10^{-C} \sum_{i=1}^{q} \sigma_{Di}^m n_{bi} + 10^{-C_d} \sum_{i=q+1}^{p} \sigma_{Di}^m n_{bi}\right)^{-1}. \quad (25)$$

The durability of the axle is determined by formulas (16), (17), (25). The calculation results for model 3 are presented in Tab. 9.

Table 9. Parameters of fatigue curves and calculation results for model 3

Characteristics	Section 1-1	Section 2-2
σ_{-1D}, MPa	150.7	175.1
m	11.53	13.01
C	31.29	35.36
C_d	33.11	37.19
M, by (25)	415731	1290624
N_Σ, cycles, by (16)	$3.008 \cdot 10^{11}$	$9.337 \cdot 10^{11}$
L_s, km, by (17)	$7.936 \cdot 10^{8}$	$2.464 \cdot 10^{9}$

6. CONCLUDING REMARKS

The design of railway car axles is usually based on ensuring their safety margin and does not finish by determining durability [21]. The paper proposes a method for determining the service life of axles, which is based on fatigue tests of samples made of their material, models of fatigue curves and the theory of similarity of fatigue failure, taking into account the real nature of loading.

The study of the stress state of the railway axle using the finite element method showed that the most dangerous are two areas of fillet transitions (sections 1-1 and 2-2, Fig. 1, *a*). The computational and experimental method has established that, according to the criterion of fatigue resistance, the axle fillet more dangerous (section 1-1, Fig. 1, *a*) due to the increased stress concentration, which leads to a significant decrease in the endurance limit in this area.

Calculations based on three well-known models of fatigue curves showed that the durability of the axles is more than 10^{10} cycles, which indicates the likelihood of their destruction in the gigacycle fatigue region.

It has been established that the durability of the axle determined for section 1-1 is 3...5 times less than for section 2-2. The calculations performed are confirmed by the known facts of destruction of axes along the axle fillet [22-24], Fig. 8.

Fig. 8. Axle fillet fracture that caused the accident at Viareggio station, Italy [22]

It is recommended to improve the geometry of fillet and apply technological surface hardening of this zone.

It can be shown that the experimentally established spectrum of axle stresses [3, 6], used to calculate the durability of the axle in this article, indicates that this operating mode belongs to the category of particularly light ones [7]. The stress spectrums presented in [4, 5] have a similar character. Therefore, to assess the durability of railway axles, it is necessary to take into account the nature of the stress spectrum.

7. CONCLUSIONS

1) Based on the results of fatigue tests of seven types of samples made of steel 45, the sensitivity coefficient to stress concentration and to scale factor ν_σ, necessary for calculating the endurance limit of the axle, was determined.
2) The endurance limits $\sigma_{-1D1} = 150.7$ MPa and $\sigma_{-1D2} = 175.1$ MPaare calculated for the two dangerous sections of the railway axle.
3) Using three well-known models of fatigue curves, the durability of the considered axle was determined. Calculations are based on the experimentally determined stress spectrum [3, 6].
4) It has been established that for the considered axle design the most dangerous zone is the area of the axle fillet 1-1.
5) For practical calculations of the service life of railway axles at the design stage, the Haibach fatigue curve model (18) is recommended, the calculation according to which gave the minimum durability value.

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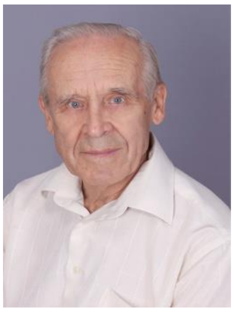

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