This study analyzes vibration signals related to bearing defects using a method that reconstructs an effective signal. This reconstruction is based on the determination of the instantaneous amplitude and phase. Then, a decomposition method is applied to the amplitude and phase to obtain several simple functions. Once the functions are obtained, an evaluation of impulsivity is performed on each function using the proposed parameter. This selects functions that contain fault data. The important signal is then identified and used. After the creation of the effective signal, filtering by a morphological operator with a structuring element is applied to improve the signal quality. Finally, in the spectrum of the absolute values of this signal, the defect can be detected from the frequency of the peaks. Signals from different databases were analyzed using the proposed method, illustrating the results in the form of high-amplitude peaks in the frequency of bearing component defects.

Keywords: amplitude value spectrum, defect detection, bearing, impulsivity evaluation

1. INTRODUCTION

Bearings are components consisting of an inner and outer ring, a cage, and a rolling element. They are generally used as links between shafts for transmitting motion and supporting loads. In the event of poor operating conditions, the bearings can fail, leading to partial or total machine failure. According to statistics, bearings are responsible for between 40% and 45% of faults in rotating machines [1], making them a strategic component. Several diagnostic and prognostic techniques are used to detect defects. These include acoustic analysis based on the analysis of acoustic emission signals [2]. Similarly, ultrasonic analysis enables the detection of defects on the basis of the transmission and reception of an ultrasonic wave through the material [2]. Moreover, vibration analysis focuses on monitoring abnormal vibrations produced by defects, whereas thermography analysis relies on defect detection based on variations in thermodynamic properties [2]. However, vibration
analysis is more feasible for fault monitoring and detection because it offers several signal processing methods [3]. Fault detection using vibration signal analysis consists of two essential steps [4]. The first step involves the collection of signals by a measurement chain comprising acceleration, speed, or displacement sensors [4]. The second step involves processing the signals using a specific diagnostic method [4]. Signal analysis is performed in three domains: the time domain, where the signal varies as a function of time; the frequency domain, which shows the variation of vibration amplitude as a function of frequency; and the time–frequency domain, which illustrates the signal as a function of both time and frequency [4].

The analysis of vibration signals in the time domain is performed using statistical indicators to assess the bearing condition, such as the statistical parameter known as kurtosis, created by D. Dyer et al. [5]. This parameter takes a value lower than three for a healthy bearing with a Gaussian distribution of vibration amplitude [5]. Many statistical indicators are used in the field of defect detection, such as the Gini index of economic origin [6]. However, it is applied in the processing of vibration signals to evaluate the pulses produced by the defect [6]. This index is more efficient than kurtosis [6]. In addition, the L2/L1 norm proposed by Jia et al. [7], the Hoyer index used by Zhao et al. [8], and negative entropy exists. Entropy, a thermodynamic concept, is used to identify the complexity of systems, but negative entropy is a parameter used in the same context as other parameters for assessing signal impulsivity [9]. In addition, the generalized logarithm penalty strengthens the signal pulses while reducing noise, demonstrating increased efficiency [10]. The complexity of signals offers additional insight for detecting faults, such as the weighted entropy index, which can be used to select disordered and ordered signals [11]. Disordered signals containing complex data with several pulses are not similar to sinusoidal signals [11]. Furthermore, comparing data from the bearing’s healthy state with that of its current state makes it easy to detect faults without locating faulty components in the bearing [12].

Frequency signal analysis is based on the representation of the signal’s vibration amplitude as a function of frequency, using transformations such as the Fourier transform to obtain the spectrum and the Hilbert transform. The equations below express the amplitude and phase of the signal [20].

\[ H[x(t)] = x(t) \ast \frac{1}{\pi t} \]  
\[ an(t) = x(t) + jH[x(t)] \]  
\[ AI(t) = \sqrt{x(t)^2 + H[x(t)]^2} \]  
\[ QI(t) = \text{arctan} \left( \frac{H[x(t)]}{x(t)} \right) \]  
\[ an(t) = AI[\cos (QI) + j\sin(QI)] \]  

In this paper, we present a method for detecting bearing defects based on vibration signal analysis. The rest of the article is organized as follows: section (2) presents the proposed method, which integrates several signal processing tools. Sections (3 and 4) illustrates the application and evaluation of the performance of the proposed method using signal processing available in databases.

2. METHODS

We propose a method for the analysis of bearing vibration signals to detect defects. The proposed method comprises three steps, as shown in the flowchart (Fig. 1).

**Step 1**: Determine the instantaneous amplitude and phase using the analytical signal obtained from the Hilbert transform. The equations below express the amplitude and phase of the signal [20].
The following equations express the real and imaginary parts of the analytic signal [20]:

\[ R(t) = x(t) = A(t) \cos(Q(t)) \]  
\[ I(t) = A(t) \sin(Q(t)) \]  

(6)  

(7)  

The vibration signal is equal to the real part of the analytic signal, as shown in Eq. (6).

Once the instantaneous amplitude and phase are obtained, the EFD method is applied to break them down into several functions (F).

Empirical Fourier decomposition allows the decomposition of a signal into multiple components (F), as shown in the following steps [21]:

- Determining the spectrum of the signal to be decomposed using the Fourier transform.

\[ X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt \]  

(8)  

- The separation limit of the spectrum is defined as follows:

If: \( 0 \leq n \leq N \) and \( f_n \neq f_{n+1} \)

\[ \omega_n = \min \{ \hat{F}(\omega) \} \]  

(9)  

If: \( 0 \leq n \leq N \) and \( f_n = f_{n+1} \)

\[ \omega_n = f_n \]  

(10)  

- The construction of band-pass filters \( u(\omega) \) is performed for each segmentation with a cut-off frequency \( (\omega_n, \omega_{n-1}) \).

\[ u(\omega) = \begin{cases} 
1 & \text{if } \omega_{n-1} \leq |\omega| \leq \omega_n \\
0 & \text{else} 
\end{cases} \]  

(11)  

- The filtered signals, denoted as \( X_n(\omega) \), originate from the decomposition of the signal to be decomposed.

\[ X_n(\omega) = u(\omega)X(\omega) \]  

(12)  

\[ X_n(\omega) = \begin{cases} 
X(\omega) & \text{if } \omega_{n-1} \leq |\omega| \leq \omega_n \\
0 & \text{else} 
\end{cases} \]  

(13)  

- The decomposed components in the time domain are obtained by applying the inverse Fourier transform.

\[ F_n(t) = \int_{-\infty}^{+\infty} X_n(\omega)e^{j\omega t} d\omega \]  

(14)  

- The sum of all the components allows for the reconstruction of the signal.

\[ x(t) = \sum_{n=1}^{N} F_n(t) \]  

(15)  

**Step 2:** Assess the impulsivity of functions (F) using statistical parameters. Kurtosis is a parameter defined by Eq. (16) [12].

\[ Ku = \frac{1}{L^4} \sum_{i=1}^{L} (x_i - \bar{x})^4 \]  

(16)  

For a healthy bearing, kurtosis is less than three, whereas for a failing bearing, kurtosis is greater than three [5]. Negentropy, Hoyer index, Gini index, and L2/L1 norm are used to assess the impulses of the fault signal, which are defined by the following equations:

- **L2/L1 norm** [22]:

\[ \frac{L2}{L1} = \frac{\sum_{i=1}^{L} x_i^2}{\sum_{i=1}^{L} |x_i|} \]  

(17)  

- **Hoyer index** (HI) [22]:

\[ HI = \sqrt{\frac{\sum_{i=1}^{L} x_i^2}{\sum_{i=1}^{L} |x_i|}} \]  

(18)  

- **Negentropy (NE)** [23]:

\[ NE = \frac{1}{L} \sum_{i=1}^{L} \left[ \frac{x_i^4}{x_i^2} \ln \left( \frac{x_i^2}{x_i^4} \right) \right] \]  

(19)  

- **Gini index** (GI) [6]:

\[ GI = 1 - 2 \sum_{i=1}^{L} \frac{x_i}{L} \left( \frac{L-i+0.5}{L} \right) \]  

(20)  

On the basis of parameter thresholds, we can determine whether the bearing is failing or healthy, as in the case of Kurtosis. However, the thresholds for the Hoyer index, Gini index, negentropy, and L2/L1 norm are unknown. Thus, we define the threshold values from the vibration signals of healthy bearings available in the CWRU database. The signals used are shown in Table (1) [24].

<table>
<thead>
<tr>
<th>Speed [rpm]</th>
<th>Load [Nm]</th>
<th>Signals</th>
<th>Signal symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1730</td>
<td>2206.47</td>
<td>100.mat</td>
<td>S4</td>
</tr>
<tr>
<td>1750</td>
<td>1470.98</td>
<td>99.mat</td>
<td>S3</td>
</tr>
<tr>
<td>1772</td>
<td>735.49</td>
<td>98.mat</td>
<td>S2</td>
</tr>
<tr>
<td>1797</td>
<td>0</td>
<td>97.mat</td>
<td>S1</td>
</tr>
</tbody>
</table>

Depending on the values of the parameters in Table (2), several thresholds differ in defining the bearing state. In this case, we propose a parameter (P) that is considered to be the product of all statistical parameters used to evaluate signal impulsivity. Parameter (P) enables the definition of a threshold and ensures the elimination of differences between parameters, as shown in the following equation:

\[ P = Ku \times L2/L1 \times NE \times GI \times HI \]  

(21)  

<table>
<thead>
<tr>
<th>Signals</th>
<th>S4</th>
<th>S3</th>
<th>S2</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>2.8658</td>
<td>2.7699</td>
<td>2.8066</td>
<td>2.8228</td>
</tr>
<tr>
<td>Negentropy</td>
<td>0.7078</td>
<td>0.6805</td>
<td>0.6948</td>
<td>0.7069</td>
</tr>
<tr>
<td>L2/L1 norm</td>
<td>1.2469</td>
<td>1.2369</td>
<td>1.2429</td>
<td>1.2478</td>
</tr>
<tr>
<td>Hoyer index</td>
<td>0.1999</td>
<td>0.1933</td>
<td>0.1972</td>
<td>0.2004</td>
</tr>
<tr>
<td>Gini index</td>
<td>0.4109</td>
<td>0.4036</td>
<td>0.4084</td>
<td>0.4126</td>
</tr>
<tr>
<td>Parameter (P)</td>
<td>0.2077</td>
<td>0.1812</td>
<td>0.1951</td>
<td>0.2058</td>
</tr>
</tbody>
</table>

According to the threshold of parameter (P), the bearing state can be defined as follows:

\[ P_{threshold} = 1.25 \times \max (P) = 0.25 \text{ (if } P \leq 0.25 \text{ healthy bearing)} \]
\[ P_{threshold} = 1.25 \times \max (P) = 0.25 \text{ (if } P > 0.25 \text{ faulty bearing)} \]

The new vibration signal, which contains fault information, is formulated as follows:

\[ \{ A\text{I}_n(t) = \sum F_{\text{A}(t)} \sum F_{Q(t)} \} \]
\[ \{ A\text{I}_n(t) = \sum F_{\text{A}(t)} \sum F_{Q(t)} \} \]

With the threshold values for the Hoyer index, Gini index, negentropy, and L2/L1 norm defined, the threshold values are applied to the vibration signals of healthy bearings to ensure the elimination of differences between parameters, as shown in the following equation:

\[ P = Ku \times L2/L1 \times NE \times GI \times HI \]  

(21)  

**Step 3:** Noise suppression by signal filtering using the MHPO1 morphological operator developed by B.Chen et al. [25].

\[ MHPO1(n) = AHDE(n) \times AHCO(n) \]  

(26)  

However, in this operation, we use a flat structuring element because it is very useful for
analyzing vibration signals and is simple. The length of a structuring element must satisfy the following condition [26]:

\[ L_{SE} < \frac{f_s}{f_c} \]  

(27)

The flat structuring element consists of zeros with a flat shape of zero height and specific length [27]. Equation (28) expresses the flat element in terms of scale (n) and length, which is equal to (n+2) [27].

\[
\{ SE_i = 0 \quad i = 1, 2, \ldots, (n + 2) \\
SE = \{ SE_1, SE_2, \ldots, SE_{n+2} \}
\]  

(28)

In this step, the length of the structuring element used is equal to 12 on a scale of 5.

Instead of using the Hilbert transform or the Teager–Kaiser energy operator as the final step in the method to demodulate the signal, determine the envelope spectrum, and then detect the fault, we develop the absolute value spectrum to identify faults by comparing theoretical fault frequencies with peak frequencies in the spectrum. The absolute value spectrum is defined as follows:

\[ x(f) = \int_{-\infty}^{\infty} |x(t)|e^{-j2\pi ft}dt \]  

(29)

3. EXPERIMENTAL STUDY

3.1. CWRU database

The CWRU database presents bearing vibration signal measurements from a test strip comprising an electric motor with two bearings [24]. One bearing is positioned on the driver side with a type 6205-SKF, and the other is positioned on the fan side with a type 6203-SKF [24]. Bearing vibration signals were collected using accelerometers mounted on bearing housings and recorded as MATLAB files [24]. Fault frequencies were calculated for each bearing component by multiplying the rotational speed in hertz (Hz) by the coefficients shown in Table 3 [24].

<table>
<thead>
<tr>
<th>Components</th>
<th>6203-SKF</th>
<th>6205-SKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner race</td>
<td>4.9469</td>
<td>5.4152</td>
</tr>
<tr>
<td>Outer race</td>
<td>3.0530</td>
<td>3.5848</td>
</tr>
<tr>
<td>Cage</td>
<td>0.3817</td>
<td>0.39828</td>
</tr>
<tr>
<td>Ball</td>
<td>3.9874</td>
<td>4.7135</td>
</tr>
</tbody>
</table>

To evaluate the effectiveness of the proposed method, we analyzed two vibration signals, one dependent on bearing 6205 and the other linked to bearing 6203. The characteristics of the signals are shown in Table 4 [24].

<table>
<thead>
<tr>
<th>Bearing</th>
<th>6203-SKF</th>
<th>6205-SKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed and Load</td>
<td>1730 RPM</td>
<td>1750 RPM</td>
</tr>
<tr>
<td></td>
<td>2237.1 Nm/s</td>
<td>1491.4 Nm/s</td>
</tr>
<tr>
<td>Defect diameter</td>
<td>0.1778 mm</td>
<td>0.1778 mm</td>
</tr>
<tr>
<td>Fault frequency</td>
<td>142.63 Hz</td>
<td>157.94 Hz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>12 kHz</td>
<td>48 kHz</td>
</tr>
</tbody>
</table>

3.1.1. Results and discussions

Case 1: Signal from the 6205-SKF bearing

The amplitude (AI) and phase (QI) of the signal are shown in Figure 2. For the EFD method, the decomposition level is four, and the amplitude (AI) and phase (QI) functions are shown in Figures (3) and (4).

<table>
<thead>
<tr>
<th>Component</th>
<th>Inner race</th>
<th>Inner race</th>
</tr>
</thead>
</table>

Fig. 1. Proposed method

Fig. 2. Amplitude and phase
According to the values of parameter (P) used to evaluate impulsivity (Table 5), the new vibration signal containing fault data can be reconstructed using the following formula:

\[ x(t) = [F_3 A_{I}(t) + F_4 A_{I}(t)] \times \cos(F_4 Q_{I}(t)) \]  

(30)

Table 5. Values of parameter (P)

<table>
<thead>
<tr>
<th>Functions</th>
<th>Phase (QI)</th>
<th>Amplitude (AI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.0398</td>
<td>0</td>
</tr>
<tr>
<td>F2</td>
<td>0.0559</td>
<td>0.0873</td>
</tr>
<tr>
<td>F3</td>
<td>0.1524</td>
<td>0.6253</td>
</tr>
<tr>
<td>F4</td>
<td>0.3003</td>
<td>3.0512</td>
</tr>
</tbody>
</table>

In the spectrum of absolute values (Fig. 5) of the new signal filtered by the MHPO1 operator, we observe a high-amplitude peak at a frequency of 157.5 Hz. This value is very close to the fault frequency of the inner ring of bearing 6205-SKF (157.5 Hz ≈ 157.94 Hz). In this case, the faulty component is identified and located at the peak frequency.

**Case 2:** Signal from the 6203-SKF bearing

The time and frequency domains of the vibration signal are shown in Fig. 6.

The frequency domain, or signal spectrum, shows several peaks at different frequencies, making fault detection impossible because of the shape complexity. To this end, we apply the proposed method to simplify the shape. The results of this method are summarized as follows:

- The amplitude (AI) and phase (QI) of the signal are illustrated in Fig. 7, and the values of parameter (P) for the four functions obtained by the EFD method are shown in Table 6. According to parameter (P), the new vibration signal, which contains data dependent on defects, is expressed as follows:

\[ A_{I}(t) = F_3 A_{I}(t) + F_4 A_{I}(t) \]

\[ Q_{I}(t) = F_3 Q_{I}(t) + F_4 Q_{I}(t) \]

\[ x(t) = A_{I}(t) \times \cos(Q_{I}(t)) \]  

(31)

(32)

Table 6. Parameter (P) values of the 6203 bearing signal

<table>
<thead>
<tr>
<th>Functions</th>
<th>Phase (QI)</th>
<th>Amplitude (AI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.0382</td>
<td>0</td>
</tr>
<tr>
<td>F2</td>
<td>0.0855</td>
<td>0.0705</td>
</tr>
<tr>
<td>F3</td>
<td>0.2864</td>
<td>0.3322</td>
</tr>
<tr>
<td>F4</td>
<td>0.2693</td>
<td>0.3562</td>
</tr>
</tbody>
</table>
3.2. Discussions and results

On the basis of the EFD method decomposition of the signal amplitude and phase into four functions (F), the impulsivity of each function is evaluated by parameter (P). Then, on the basis of the values of parameter (P), the new signal is reformulated as defined by Eqs. (24 and 25). The main results are illustrated as follows:

- The new vibration signal is expressed as follows:
  \[
  A_I(t) = \sum_{i=2}^{4} F_i(t) \\
  Q_I(t) = F_4(t) \\
  x(t) = A_I(t) \times \cos(Q_I(t))
  \]

3.2.1. Ottawa university database

The vibration signals in this database were collected using a test strip consisting of a single-phase electric motor operating at a constant speed of 1750 rpm [28]. The motor shaft is supported by model 6203 ZZ ball bearings, and vibrations from these bearings are collected by a model PCB 623C01 accelerometer mounted by a magnet on the bearing housing [28]. Vibration signal data were connected between the computer and sensor via a data acquisition system and recorded as MATLAB files with a sampling frequency of 42 kHz [28]. In addition, signals related to inner ring, outer ring, and cage faults were recorded under a load of 400 N, but no load was applied to signals related to ball faults [28].

We analyzed the fault signal of the outer ring of the NSK 6203 ZZ bearing according to Eq. (33) [11] and the parameters in Table 7 [28]. The fault frequency of the outer ring is 88.95 Hz.

\[
F_{or} = \frac{2\pi \nu F_r}{2m} \left( 1 - \frac{d}{D_m} \cos(\alpha) \right) \quad (33)
\]

The spectrum of the absolute values of the filtered signal reveals a high amplitude peak at the outer ring fault frequency (88.44 Hz ≈ 88.95 Hz), as shown in Figure 9. Thus, the fault is identified at the frequency of the peak.

4. DETECTION METHOD COMPARISONS

Several diagnostic methods exist, but the most popular and widely used are envelope analysis and the deconvolution method. The envelope analysis consists of two steps [29]:

- Step 1: Filtering the vibration signal using a band-pass filter centered on the resonance frequency. This frequency is determined using methods such as Kurtogram and Autogram.
- Step 2: Determination of the envelope spectrum using the Fourier and Hilbert transforms, as

<table>
<thead>
<tr>
<th>Functions</th>
<th>Phase (QI)</th>
<th>Amplitude (AI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.0299</td>
<td>0</td>
</tr>
<tr>
<td>F2</td>
<td>0.1537</td>
<td>0.4313</td>
</tr>
<tr>
<td>F3</td>
<td>0.1558</td>
<td>0.3634</td>
</tr>
<tr>
<td>F4</td>
<td>0.5752</td>
<td>2.7055</td>
</tr>
</tbody>
</table>

Table 8. Evaluation of impulsivity by parameter (P)
described in equations (3) and (35). Next, the fault is detected by analyzing the peak frequency.

\[ AI(f) = \int_{-\infty}^{+\infty} A(t)e^{-j2\pi ft} dt \]  

(35)

Thus, deconvolution methods such as MCKD are based on the extraction of signal pulses [11]. Then, a determination of the signal envelope spectrum obtained after the deconvolution operation is performed to detect the defect [11]. The steps of these methods are as follows [11]:

- Step 1: Deconvolution of the signal using the MCKD method.
- Step 2: Identify the envelope spectrum using equations (3) and (35).

The envelope analysis and fault detection approach using deconvolutions are applied to the outer ring vibration signal present in the University of Ottawa database, which was previously analyzed using the proposed method. Once the envelope analysis has been performed, a kurtogram of the signal illustrates a maximum value of spectral kurtosis (48.2), as shown in Figure (10). The two pass frequencies of the filter are 10500 Hz and 15750 Hz, with a resonant frequency of 13125 Hz and a bandwidth of 5250 Hz. Then, in the envelope spectrum, we find a significant peak at the fault frequency (88.44 Hz ≈ 88.95 Hz), as shown in Figure (11).

4.1. Influence of decomposition level

The proposed method involves two parameters: the vibration signal and the number of functions to be decomposed using the EFD method. The effect of the number of functions on the final result is visualized according to the frequency of the peak present in the spectrum of absolute values, for each variation of the number of functions (F) between 2 and 5. After applying the proposed method to the vibration signal for a variation in the number of functions from 2 to 5, the new vibration signals are shown in Table (9).

In the spectrum of absolute values of each new signal, decomposition levels 2, 3, 4 and 5 show better results. This is illustrated in Figures (13 and 14). Increasing the decomposition levels from five, such as 6 and 10, shows the same results, indicating that the decomposition level has no influence.

<table>
<thead>
<tr>
<th>Decomposition level</th>
<th>New signals</th>
</tr>
</thead>
</table>
| 2                   | \[ AI_n(t) = F_2(t) \]
|                     | \[ QI_n(t) = F_2(t) \]
|                     | \[ x(t) = AI_n(t) \times \cos(QI_n(t)) \] |
| 3                   | \[ AI_n(t) = F_2(t) + F_3(t) \]
|                     | \[ QI_n(t) = F_2(t) \]
|                     | \[ x(t) = AI_n(t) \times \cos(QI_n(t)) \] |
there are differences in the selection of effective functions (F), determined by the EFD method. To avoid this difference, we propose a parameter (P) considered as a product between all parameters. Finally, we obtain a single important threshold that facilitates the reconstruction of an effective signal.

The spectrum of absolute values enables faults to be detected simply and efficiently, based on peak frequency. It can be used as the final diagnostic step.

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