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RESEARCH OF STRESS CONCENTRATION AT CLOSELY PLACED HOLES IN WING BEARING AREA IN ANISOTROPIC PLATES

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Abstract

In this article effective approach of the study of high-stress concentration at closely placed holes in wing bearing area (in anisotropic plates) is proposed. It is based on the boundary integral equation method with the additional use of the asymptotic method. The simplicity, precision of the approach and the stability of the solution are illustrated in the calculation of stresses in the plate with a circular hole, an elliptical hole, elongated holes, a plate with two closely spaced elliptical holes

Keywords: stresses; composite materials; stress concentration, boundary integral equations method; plates with holes; wing bearing area

List of Symbols/Acronyms

BIEM – boundary integral equation method;
SSS – stress-strain state;
SCF –stress concentration factors;
LSM – least squares method;
SIF – Stress Intensification Factors

1. INTRODUCTION

High strength properties, low specific weight, resistance to the action of aggressive environments led to the wide use of composite materials in aviation technology. In heavy aviation, auxiliary structures are mostly made from composite materials. In light aviation, especially unmanned 1st and 2nd class - this is a glider design. In mechanical engineering and other industries, composite structural elements weakened by hole systems are also widely used. The assessment of the strength and durability of composite structural elements is based on the analysis of their stress-strain state (SSS) with full consideration of the anisotropy of mechanical and strength characteristics [5]. To study the SSS of isotropic and anisotropic plates with holes, the method of boundary integral equations (BIEM) is widely used [3, 14, 19].

Using this method, stresses near holes of different shapes in composite plates were studied [1, 2, 4, 15, 20, 22]. In composite elements of aviation equipment, a system of holes is often created. In particular, the rigid skin of aircraft has closely spaced holes in the places of its attachment to the power frame of the aircraft. Cladding is made from separate sheets or panels of different types of materials. BIEM also proved to be effective for calculating stresses in such plates taking into account the interaction of holes [6, 7, 21].

When considering closely spaced holes, stress concentration coefficients (SCF) increase and can become infinite [9, 16, 18]. Therefore, direct numerical methods of studying stresses for such openings, which were used in works [2-4], are ineffective. For two circular holes in isotropic plates, asymptotic formulas for the SCF at close-to-zero distances between them have been established [18]. Based on these formulas, [18] proposed an approach in which the structure of the formula for determining the SCF at small distances containing unknown steels is pre-established. These steels were determined based on the calculated SCFs using BIEM for selected distances followed by the method of least-squares. It was shown in [21] that this approach is also effective when considering isotropic plates with holes of a different shape (in particular, elliptical). In this work, a similar approach is proposed for the study of stresses near closely spaced holes in composite plates. To do this, we first proposed an asymptotic formula for stresses near holes in anisotropic plates. The implementation of the approach was carried out with the combined application of the method of integral equations [10-13, 18] and the method of least squares. At the same time, simple relations for determining the SCF were obtained, which turned out to be practically accurate for a wide range of distances between the holes - both infinitely small and commensurate with the radius of the hole.

2. FORMULATION OF THE PROBLEM

It is considered an orthotropic plate (Fig. 1) with two identical elliptic holes with a semiaxes, b, whose centers are at the points $(-a-d, 0), (a+d, 0)$ *u*nder stretching of the plate in the direction of *Oy* axis with tractions p. At close distances between the holes, the concentration of stresses around the points $(\pm d, 0)$ increases quickly, and therefore, for calculations with controlled accuracy, it is necessary to increase the number of nodal points.

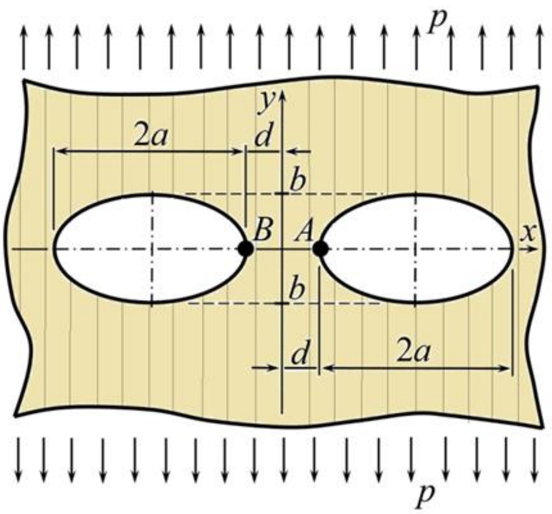

Fig. 1. Scheme of loading the plate with a holes

To illustrate the difficulties encountered in the direct application of BIEM, the studying of the influence of the number of nodal points N in the quadrature method on the accuracy of calculations of maximum stresses on the contour at short distances between the holes was performed. The stress state was calculated on the basis of the integral representation [11-13], which takes into account the symmetry of the problem about the *Oy* axis and the integral equations are written only at the boundary of the right hole.

A detailed study was performed for two composite materials CF1 (carbon fiber reinforced plastic) and EF (reinforced epoxy phenolic plastic) [23,24].

Modules of elasticity, Poisson's ratios, shear modulus for these material given in table1 [23].

Table 1. Elastic constants

Material	E_1 (GPA)	E_2 (GPA)	G_{12} (GPa)	ν_{21}	ν_{12}
CF1	8.62	400	2.8	0.35	0.007
EF	21	32.8	5.7	0.21	0.134

The first of these materials belongs to the class of highly anisotropic, and the second to slightly anisotropic.

Table 2 shows the values of the maximum hoop stresses σ_θ divided by p (it are indicated by the symbol $s_{\max}$ ($s_{\max} = \sigma_{\theta\max} / p, \sigma_{\theta\max} = \max(\sigma_\theta)$) for the case of circular holes in isotropic plates at different values of dimensionless distance $D = d/a$.

Table 2. The accuracy of calculations of stresses of isotropic plate

D	0.001		0.01		0.1
N	$s_{\max}$	$\tilde{s}_{\max}$	$s_{\max}$	$\tilde{s}_{\max}$	$s_{\max}$
100	143.043	100.39	20.733	20.463	6.106
200	131.519	76.387	19.658	20.363	6.106
300	114.989	69.825	20.131	20.363	–
400	103.956	68.249	20.312	20.363	–
500	96.082	67.808	20.353	20.363	–

The performed calculations showed that high stresses occur only in the vicinity of the semi axes of the ellipse placed on the *Ox* axis. In such cases, the parametric giving of the contour is modified for improve efficiency BIEM. Let's write the boundary contour equation in parametric form as $x = \phi(\theta), y = \psi(\theta)$, where $0 < \theta \leq 2\pi$. In particular, at considering elliptical holes, giving $\phi = a\cos\theta, \psi = b\sin\theta$ was used. Modified givings are written as $x = \tilde{\phi}(\tau), y = \tilde{\psi}(\tau)$ where $\theta = g(\tau)$, $0 < \theta \leq 2\pi$. The function $g(\tau)$ is chosen in such a way that it is monotonically increasing and that the derivative of it was small in regions with maximum stress concentration. With this choice, in these areas, the increase of density of nodal points will be there, which leads to an increase in the accuracy of calculations. The relative stresses thus calculated at [16] $g(\tau) = \tau - c\sin 2\tau$, $c=0,475$ are shown in Table and are indicated by $\tilde{s}_{\max}$. Similar results for the anisotropic plate made of CF1 material with stretch in the direction of greater and less stiffness of the material are shown in Tables 3 and 4, respectively.

Table 3. The accuracy of calculations of stresses in a plate made of material CF1

D	0.001		0.01		0.1
N	$s_{\max}$	$\tilde{s}_{\max}$	$s_{\max}$	$\tilde{s}_{\max}$	$s_{\max}$
100	262.941	226.578	55.201	55.352	20,996
200	232.226	225.046	55.189	55.313	20,996
300	226.869	224.424	55.256	55.312	–
400	225.484	224.192	55.291	55.312	–
500	224.778	224.104	55.305	55.312	–

Table 4. The accuracy of calculations of stresses in a plate made of material $CF1_{90}$

D	0.001		0.01		0.1	
N	$s_{\max}$	$\tilde{s}_{\max}$	$s_{\max}$	$\tilde{s}_{\max}$	$s_{\max}$	$\tilde{s}_{\max}$
100	132.853	65.374	18.401	19.203	5.928	6.004
200	106.956	64.602	17.963	19.365	6.001	6.017
300	84.388	63.882	18.247	19.479	6.015	6.018
400	71.953	63.686	18.516	19.535	6.018	6.018
500	64.463	63.743	18.736	19.563	6.018	6.018

The data in the Tables shows that a large number of nodal points must be selected to ensure the controlled accuracy of calculations using the mechanical quadrature method. In particular, at relative distances *d/R~0.01*, using the direct BIEM method, it is necessary to select up to 500 nodes, and at *d/R~0.001* it is necessary to solve systems ~ 1000-4000 equations.

Tables 1-3 show that nonlinear transformations of the parameter allow reducing the number of nodal points when calculating the maximum stresses with controlled accuracy. It should be noted that a similar method is used in the finite element method, when the element sizes are reduced in regions with increased stress concentration.

Nonlinear parameter transformations significantly increase the accuracy of the calculations.

3. ASYMPTOTIC METHOD OF INVESTIGATING OF STRESSES NEAR CLOSELY SPACED HOLES

The complexity of the study of stress concentration is that with zero distance between the holes, the stresses is infinite. In [16], it is established that the maximum stresses in isotropic plate at $d \to 0$are of the order $\sigma_{\theta\max} \sim C/\sqrt{D}$, where C is constant. Assume that in anisotropic plates $\sigma_{\theta\max} \sim C/D^m$, m is a constant that must be determined.

Let us further introduce the relative hoop stress $\sigma = \sigma_{\theta\max}/\sigma_0$, where σ_0 is the maximum stress on the single hole for the chosen load. With small distances between the holes the relative stresses are determined in the form $\sigma = f(\delta)$ [16], where

$$f(\delta) = \frac{a_1}{\delta^m} + a_2 + a_3\delta^m, \qquad (1)$$

where $\delta = \frac{D}{1+D}$, m, a_j are constants, $j = 1,2,3$.

The constants are determined by least squares method (LSM) with conditions is the minimum.

$$I = \sum_{j=1}^{M}\left[f(\delta_j) - \sigma_j \right]^2, \qquad (2)$$

Herein δ_j are the sequence the parameter's value δ, that were selected at interval $\delta_{\min} < \delta < \delta_{\max}$, σ_j are the maximum relative stress values at a relative distance δ_j.$m = 0{,}5$[18] is obtained for isotropic plates. For anisotropic plates, this constant was obtained by *a* successive approximations in the least-squares method. The performed calculations showed that based on the criteria (2) are enough precise determined of the constants m, a_1. Other constants are more precisely obtained by minimization of value.

$$I_* = \sum_{j=1}^{M}\delta_j^{2m}\left[f(\delta_j) - \sigma_j \right]^2, \qquad (3)$$

where the limited values are under the sign of the sum.

Let's give the value of stresses at the points of the boundary *A(a,0)*, *B(0,b)* of a single elliptical hole under the stretching of the orthotropic plate to infinity, by which the value is determined σ_0 [8]:

$$\sigma_\theta(A)/p = 1 + \left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right)\frac{a}{b}, \quad \sigma_\theta(B)/p = \beta_1\beta_2, \qquad (4)$$

where the plate is stretched in the direction of the *Oy* axis with tractions *p*;

$$\sigma_\theta(B)/q = 1 + (\beta_1 + \beta_2)\frac{a}{b}, \quad \sigma_\theta(A)/p = \frac{1}{\beta_1\beta_2}, \qquad (5)$$

where the plate is stretched in the direction of the *Ox* axis with tractions *q*. Here $\beta_{1,2} = \mathrm{Im}(s_{1,2})$.

4. RESULTS OF THE CALCULATIONS

At calculations we accepted $\delta_{\min} = 0{,}0004$, $\delta_{\max} = 0{,}5$. At minimum distances in the calculations, we chose up to *2000* nodal points. The calculated value $F = \delta^m \sigma_{\theta\max}/\sigma_0$ for the isotropic plate at the ratio of the semi axis *b/a=1*(circular hole) and *b/a=1/2* depending on $\ln\delta$ is shown in Fig. 2.

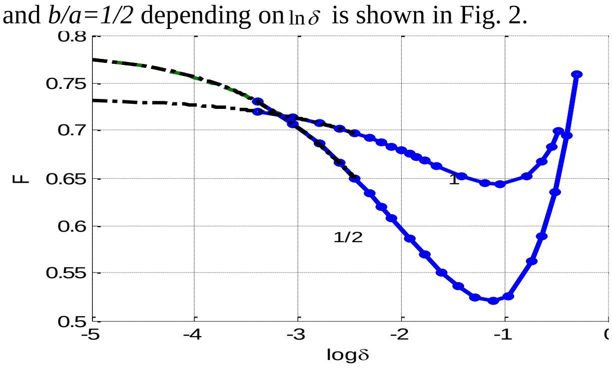

Fig. 2. The relative maximum stress in isotropic plate, 1 - circular holes, 1/2 - elliptical holes

The relative maximum stress which are calculated by BIEM are depicted by circles, those ones which are obtained by the formula (1) are depicted by the curves– after using the least squares method for obtaining the coefficients a_1, a_2, a_3. The ratio of the semi-axes of the elliptical hole b/a is indicated near the curves. On the interspace $\delta_{\min} < \delta < \delta_{\max}$, formula (1) belongs to the class of interpolation, since it is obtained on the basis of almost exact values of stresses on this interspace (depicted in Fig. 2 by circles). In Figure 2 and below, the stresses obtained on this interspace according to formula (1) are depicted by solid lines, and on the interspace $0 < \delta < \delta_{\min}$ by dashed lines.

The coefficients a_1, a_2, a_3 determined by least-squares method are shown in Table5. The values of these coefficients at $b/a = 1$ was obtained in [16] are equal to $a_1 = 0{,}7330, a_2 = -0.6497$, $a_3 = 0.8127$ (here the difference in the factors in [16] near the coefficients is taken into account).

Table 5. Table of coefficients of formula (1) for isotropic material

b/a	*m*	a_1	a_2	a_3
1	0.5	0.734	-0.717	0.732
1/2	0.5	0.783	-2.826	0.774

We can see that the first coefficient almost coincides with the obtained value by us *0.7338*. At *b/a = 0.5*, the first coefficient obtained in [16] is equal to $a_1 = 0{,}786$ which also coincides with the obtained value by us *0.7831*. The shown in Fig. 2 curves are close to the obtained stress distribution in [16] by another method.

Similar results are obtained for anisotropic materials. In plate made of material CF1 the obtained power factor is equal to $m = 0.6338$. The results of the relative stress calculations are shown in Fig. 3.

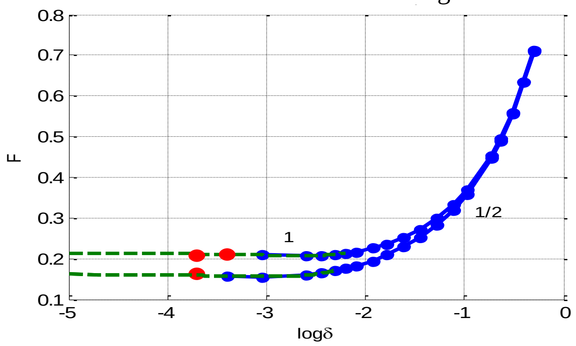

Fig. 3. The relative maximum stress in a plate made of material CF1, 1 - circular holes, 1/2 - elliptical holes

The constants *m* and the coefficients a_1, a_2, a_3 of formula (1) are given in Table 6.

Table 6. Table of coefficients of formula (1) for material CF1

b/a	*m*	a_1	a_2	a_3
1	0.634	0.214	-0.662	-0.214
½	0.634	0.162	-0.845	0.161

We considered case where the direction with the bigger stiffness of CF1 material is parallel to the *Ox* axis. The results of the calculations are shown in Fig. 4 and Table 7.

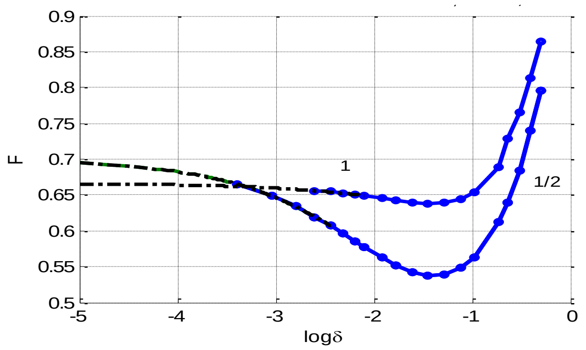

Fig. 4. The relative maximum stress in a plate made of material $CF1_{90}$; 1 - circular holes, 1/2 - elliptical holes

Table 7. Table of coefficients of formula (1) for material $CF1_{90}$

b/a	*m*	a_1	a_2	a_3
1	0.5146	0.668	-0.281	0.667
½	0.5146	0.700	-2.071	0.695

In addition, the stress at small distances on the interspace $0 < \delta < \delta_{\min}$ was calculated by BIEM. In Fig. 3 calculated relative stresses are represented by circles (red), which are almost exactly located on the curves indicated by dashed lines. That is, formula (1), which is an extrapolation for this interspace, allows you to obtained the stress with close to zero distances between the holes with high accuracy.

The calculated relative stresses and coefficients of the extrapolation formula for the material EF are shown in Figs. 5-7 and in Tables 8-10 for cases where the direction with bigger stiffness of the material EF is parallel to the axis *Oy*, *Ox* and the parallel line inclined at an angle of 45^0 respectively.

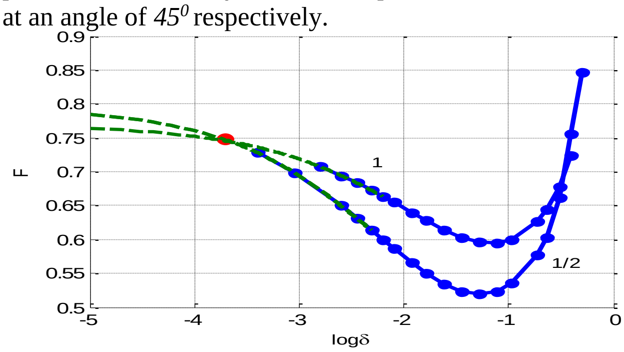

Fig. 5. The relative maximum stress in a plate made of material EF; 1 - circular holes, 1/2 - elliptical holes

Table 8. Table of coefficients of formula (1) for material EF

b/a	*m*	a_1	a_2	a_3
1	0.5	0.769	-1.762	0.764
½	0.5	0.796	-3.724	0.784

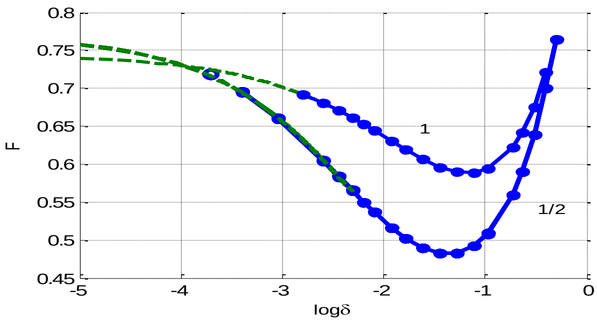

Fig. 6. The relative maximum stress in a plate made of material EF_{90}; 1 - circular holes, 1/2 - elliptical holes

Table 9. Table of coefficients of formula (1) for material EF_{90}

b/a	*m*	a_1	a_2	a_3
1	0.504	0.744	-1.487	0.739
½	0.504	0.770	-4.262	0.757

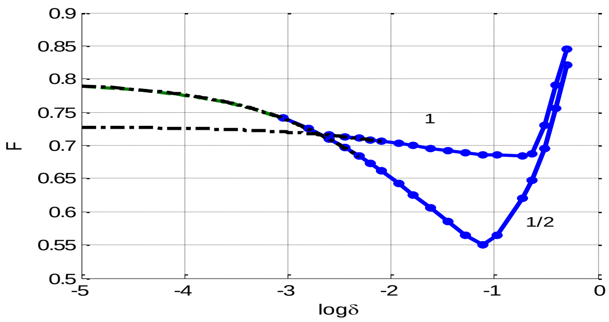

Fig. 7. The relative maximum stress in a plate made of material EF_{45}. 1 - circular holes, 1/2 - elliptical holes

The results of calculations for the isotropic plate and for plates made of materials CF1 and EF for elongated ellipses at *b / a = 2* are shown in Fig. 8.

Table 10. Table of coefficients of formula (1) formaterialEF45

b/a	*m*	a_1	a_2	a_3
1	0.495	0.730	-0.315	0.729
1/2	0.495	0.795	-1.907	0.789

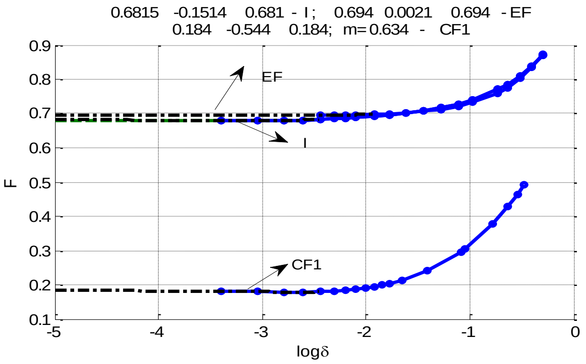

Fig. 8. The relative maximum stress in a plate with elongated ellipse holes at *b/a=2*

5. STRESS AT DISTANT POINTS AT THE BOUNDARY OF THE HOLES

Let us give the values of the divided by *p* stresses at distant points on the holes – points $(\pm(2a+d),0)$that are obtained at relative distances$\delta = 0{,}0004$ and correspond to the case of practical contact of the holes. The calculated stresses for different materials with the ratio of the semi axes *b/a* equal to *1, 0.5* and *2* are shown in Table 11 above the line.

Table 11. Relative stresses at distant points in the hole

b/a	I	CF1	$CF1_{90}$	EF	EF_{90}	EF_{45}
1	3,860	18.170	4.029	4.181	4.931	3.265
	3.829	18.651	4.002	4.167	4.958	3.451
0,5	6.619	35.044	6,187	7.337	8.7102	5.450
	6.657	36.303	6.182	7.334	8.917	5.993
2	2,407	9.782	3.090	–	–	–
	2.414	9.826	2.295			

The exact value of the relative stresses in the isotropic plate with two circular holes that touch is equal to *3,861* and it agrees well with the value given in the table.

Assume that a plate has two closely placed elliptical holes with semi axes *a, b.* The minimum radius of curvature of these ellipses *is* equal to $\rho = b^2/a$. Let's consider an equivalent ellipse with the semi axes a_1, b_1 at $a_1 = 2a$. We choose the half axes b_1 so that its minimum radius of curvature is also equal to ρ. From here we obtained $b_1 = \sqrt{2}b$. Thus, the ratio of the semi axes of the equivalent ellipse will be $a_1/b_1 = \sqrt{2}a/b$.

For the holes considered above, for which *b/a=1,0.5,2,* the minimum radii of curvature of the equivalent ellipse will be: $\rho = a, 0.25a, 4a$.

The obtained relative stresses in the equivalent ellipse are shown in Table 11 under line. It can be seen that the stresses for both isotropic and anisotropic materials at distant points at tangent holes can be calculated based on consideration of an equivalent ellipse. A slightly larger discrepancy is in the case of asymmetrical locations of the holes relative to the orthotropic axes (see last column). Similar data to the data in Table 11 is obtained for isotropic plates in [16].

6. HALF-PLANE WITH A CIRCULAR HOLE

Consider a half-plane $y > 0$ with a circular hole of radius R, the center of which is located at a point $(0, R+d)$ at stretched in the direction of the axis Ox by the forces of p. The algorithm for determining the stresses for such a plate is given in [13,17]. The formula for describing the maximum stresses near the hole at small ion distances to the half-plane boundary was also described by formula (1). The relative stresses for the isotropic (izo) plate and the plate made of EF material, depending on the value, are calculated $\delta = d / R$ shown n Fig. 9 and Table 12.

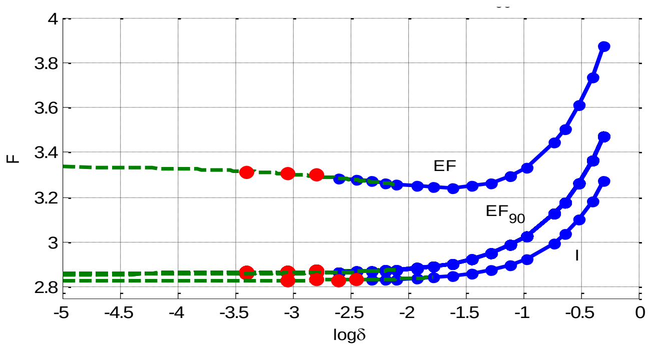

Fig. 9. Relative maximum stresses in the half-plane with a circular hole (EF material end isotropic material)

Table 12. Table of coefficients of formula (1) for half-plane with a hole

	m	a_1	a_2	a_3
Izo	0.5	2,830	-0/015	2,830
EF	0.508	3,339	-1,368	3,335
EF90	0.504	2,860	0.039	2.860

Note that for anisotropic plate at short distances, the extrapolation formula proved to be valid

$$\sigma_{\theta\max} / p = 2{,}83\left(\sqrt{\delta} + 1/\sqrt{\delta}\right) - 0{,}0154 .$$

A detailed half-plane with a circular hole at *d / R = 0.01* is considered. Calculated by the integral equations method, the stress concentration factor at the hole boundary for isotropic material is equal to SCF = 28.5676 (28.3739). In parentheses is given value SCF, which was obtained by formula (1) and table 12.

Similar results for plates made of EF when the modulus of elasticity is maximum E_x we have SCF = 33.5712 (33.7551) and SCF = 29.4118 (29.2846) at the maximum E_y .

At holes close to the half-plane boundary, large stresses arise along its boundary. Figure 10 shows the values of the stresses divided to tractions *p*.

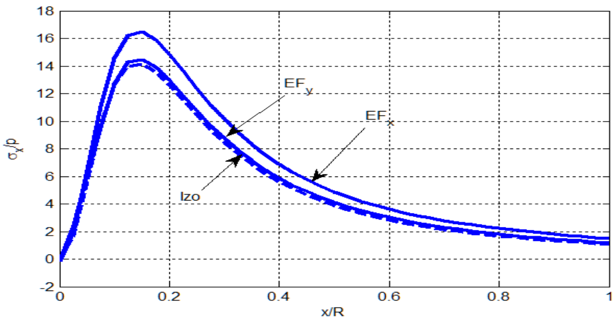

Fig. 10. Stress at the boundary of the half-plane (EF material end isotropic material)

Maximum stresses values are reached at $x = \pm 0.15d/R$.

7. STRESS CONCENTRATION AT REMOTE HOLES IN ANISOTROPIC PLATES

Relationships (1) describe quite accurately the stress distribution at $0 < \delta < 0,01$. At higher values of the parameter to determine the relative stresses $\sigma = \sigma_{\theta\max} / \sigma_0$ approximate formula used $\sigma = S(\delta)$, where

$$S(\delta) = (C_0 + C_1\delta + C_2\delta^2 + C_3\delta^3)/\delta^m . \quad (6)$$

The coefficients obtained by the method of least squares in this formula for the above cases are shown in Table 13 for composite (EF,CF1) and isotropic (Izo) materials.

Table 13. Coefficients of the formula (6)

Mat	b/a	C_0	C_1	C_2	C_3	Range, m
Izo.	1	0.686	-0.962	5.759	-8.319	$0,064 < \delta < 0,5$
	0.5	0.599	-1.446	7.743	-8.494	$m = 0.5$
CF1	1	0.207	1.823	-3.086	2.927	$0,064 < \delta < 0,5$
	0.5	0.113	1.854	-2.799	2.594	$m = 0,6338$
$CF1_{90}$	1	0.638	-0.048	2.332	-2.682	$0,064 < \delta < 0,5$
	0.5	0.565	-0.457	4.360	-5.092	$m = 0,51457$
EF	1	0.637	-0.996	6.247	-8.122	$0,012 < \delta < 0,4$
						$m = 0,5$
EF_{90}	1	0.630	-0.913	5.915	-7.687	$0,012 < \delta < 0,5$
	0.5	0.512	-0.49	4.632	-5.319	$m = 0.504$
EF_{45}	1	1.569	1.956	5.060	-7.285	$0,008 < \delta < 0,5$
	0.5	0.291	1.171	-1.306	0.273	$m = 0,5337$
Izo	2	0.686	0.605	-0.946	0.945	$0,064 < \delta < 0,5$
CF1	2	0.175	1.790	-4.152	4.906	$0,064 < \delta < 0,5$
$CF1_{90}$	2	0.694	0.350	0.009	0.002	$0,064 < \delta < 0,5$

For the purpose of testing, calculations of stresses near two holes in an isotropic plate were performed using the finite element method by the Ansys system. The shape and dimensions of the plate are shown in Fig. 10. The calculated maximum stresses at $\frac{d}{R} = 0.1$ when the plate is stretched by

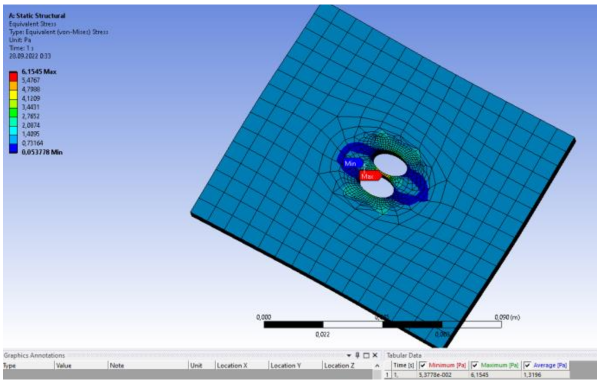

Fig. 10. Relative maximum stresses in the plane with a two circular hole (isotropic material)

tractions *p=1* are $max(\sigma_\theta) = 6.15$, which is close to the value we obtained of 6.102 (Table 1). To achieve this result, it was necessary to reduce the size of the elements near the holes (8320 elements were chosen). For a smaller value of $\frac{d}{R} = 0.01$, we were unable to achieve stability of the results of the solution with the version of the Ansys program available to us. At that time, when the stresses were investigated in calculations using the integral equations we selected, the stresses were obtained with controlled accuracy at an arbitrary distance between the holes.

8. CONCLUSION

In the article, based on the boundary integral equation method with the additional use of the asymptotic method, a high concentration of stresses near closely spaced holes in the wing bearing area is investigated ground. Known modified nonlinear parametric contour givens were used to increase the convergence of the solution at small distances between the holes. The results of calculations for isotropic plates are close in nature and magnitude to the values obtained by V.V. Panasyuk and M.P. Savruk [16]. Thus, simple formulas were obtained to calculate the SIF, which proved to be practically accurate for a wide range of distances between holes, both infinitesimally small and commensurate with their sizes.

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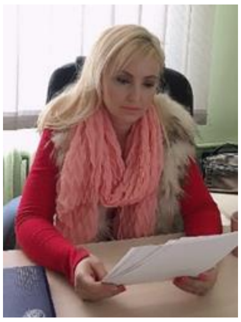

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