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THE MODAL ANALISIS OF A TWO-LINK MECHANICAL SYSTEM OF A ROBOT MANIPULATOR

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Abstract

The method of calculation of natural frequencies and forms of oscillations of a two-link mechanical system of a robot manipulator is proposed. Links of the system are considered as straight rods with a step change of cross-sectional parameters. The equations of motion of a mechanical system are based on the technical bending theory. The analysis of oscillation processes is carried out using the matrix method of initial parameters.

Keywords: mechanical system, link sections, robot manipulator

1. INTRODUCTION

Nowadays, robot manipulators, the mechanical systems of which consist of hinged links, are widely used, in mechanical engineering, transport, and other industries [1-11]. The flat or spatial crank mechanism formed in this way simultaneously serves both for change of height of lifting of freight, and for adjustment of the offset and provides high operational efficiency of technical equipment. Besides, cranked lifting devices are characterized by compactness and ease of transportation and are easily brought into working position. The task of raising the technical level and efficiency of robotic systems, which is relevant for the industry, is directly related to the need to improve the methods of their calculation. That is why the study of the peculiarities of the operation of industrial robots, in particular, the issues of kinematics and dynamics of their mechanical systems is given considerable attention in the literature [2-13].

As is known, the load-bearing structures of industrial robots during operation undergo significant dynamic loads due not only to the transient modes of operation of the drive systems, but also the action of the overhang vibration equipment.

Traditionally, the load-bearing structure of the robot is considered as a mechanical system with a finite number of degrees of freedom. Based on this approach, the condition of resistance of a multi-link robot manipulator to failures associated with the locking of hinge joints is substantiated [2]. The stability of the mechanical system is ensured by changing the configuration of the velocities of long elements based on the solution of the optimization problem. Increasing the dynamic load of a two-link robot manipulator is achieved through the use of a special controller that optimizes the control of the technical object [3]. In the work [4], the joint work of the system of parallel mechanisms of the forging manipulator is considered, and the influence of the compliance of these mechanisms on the dynamic forces in the components of the mechanical system and on the strength of parts and joints is investigated.

In the work [5], the initial position of the manipulator is optimized in order to avoid a sudden change in the hinge joint velocity. The problem of optimum design of the 5R symmetrical manipulator with minimization of a workspace is considered in the work [6]. The solution of a similar problem for parallel spatial manipulators with identical kinematic chains is given in the work [7]. In the work [8], nonlinear tracking control of kinematically redundant robot manipulators based on the use of neural networks is performed. The example of a 3R planar robot manipulator illustrates the stability of the system in the sense of Lyapunov, as well as the efficiency of the control system. In the work [9], a mathematical model of the robot mounted on a wheeled platform was constructed, and a theoretical analysis of its oscillations as a nonholonomic mechanical system was performed, taking into account the moving boundary conditions.

In order to increase the accuracy of dynamic analysis of mechanical systems of robot manipulators, continuous-discrete calculation models are developed [10-13]. On the Based of such an approach, the mathematical model of oscillation processes was constructed in the work [10], and the problem of optimization of planning the executive body trajectory of the robot manipulator taking into account large deformations of bearing elements was considered. In this study, one of the links of the mechanical system is considered as absolutely rigid, and the other is considered as flexible. The mathematical model of oscillation phenomena in a two-link mechanical system of a robot for task control was built using the Hamilton principle [11]. In the work [12], the method of generalized displacements was used to build a mathematical model of oscillations of a mechanical system of a robot with a long flexible component. According to this method, the unknown functions describing the displacement of a long flexible component and simultaneously dependent on the spatial coordinate and time are given as the sum of the products of time-dependent coefficients which perform the function of generalized coordinates, and basic functions that depend on the spatial coordinate and satisfy the boundary conditions. Features of mathematical modeling of longitudinal oscillations of long components of the robot manipulator are considered in the work [13].

The practical significance of the results of studies of the mechanical system dynamics of robot manipulators mainly lies in obtaining information about the accumulation of internal damage in the material and the residual life of structural components [14, 15]. Often the crucial factor in ensuring the proper accuracy of dynamic load determination is taking into account the dynamic properties of drive motors [16] or taking into account the interaction of a mechanical system of a robot with a load-bearing structure [17].

In order to avoid resonant phenomena in a mechanical system of a robot, as well as to build a correct mathematical model of non-stationary dynamic processes, there is a need for a modal analysis of a mechanical system of a robot. This article is aimed at solving this problem.

2. MATHEMATICAL MODEL OF A TWO-LINK MECHANICAL SYSTEM OF A ROBOT MANIPULATOR

Let us construct a mathematical model of free oscillations of a two-link mechanical system of a robot manipulator in the vertical plane. The links of the system are mostly made in the form of thin-walled or rod long structures of variable cross section joined with hinges. To simplify the calculation, we consider links with some approximation as straight rods with a step change of cross-sectional parameters. Since the lengths of the links are much larger than the sizes of their cross sections, we will take into account only the bending deformations of long elements. It is natural to assume that the tensile-compressive and shear deformations of the links have slight effect on the characteristics of the deformed state, as well as on the natural frequencies and forms of the mechanical system oscillations.

The calculation scheme of the mechanical system of a robot manipulator is shown in Fig. 1,

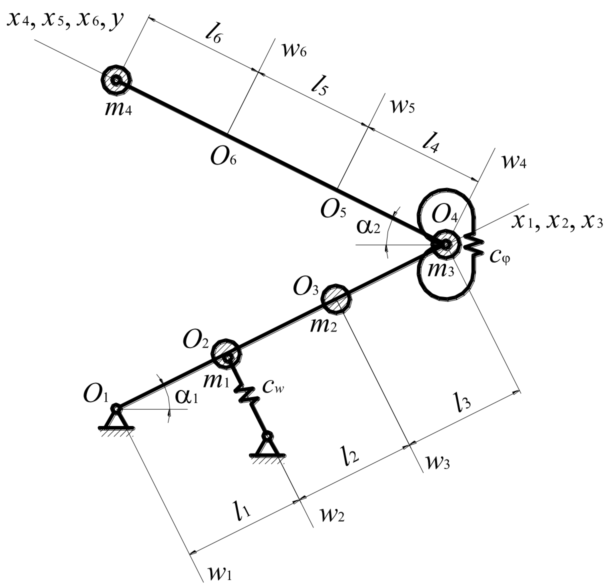

Fig. 1. Calculation scheme of a two-link mechanical system of a robot manipulator

where l_1, l_2, ..., l_6 are the lengths of link sections, within which the cross-sectional parameters are considered constant; m_1, m_2, and m_3 are masses of fastening joints, determined taking into account the consolidated masses of hydraulic cylinder components; m_4 is the mass of the clamping device with the freight; c_w is the stiffness of the hydraulic cylinder for lifting the lower link, which corresponds to the displacement of the supporting joint in the direction perpendicular to the axis of the link; c_φ is the stiffness of the hinge joint in the rotational direction; α_1 and α_2 are angles of inclination of the link axes to the horizontal; x_1, x_2, ..., x6 are longitudinal coordinates of link sections, the beginnings of which are located at points O_1, ..., O_6, respectively; w_1, ..., w_6 are flexures of link sections; y is the displacement of the upper link of the mechanical system in the longitudinal direction.

The equation of transverse oscillations of section parts of the mechanical system is written using the technical bending theory

$$\frac{\partial^4 w_i}{\partial \xi_i^4} + \frac{\mu_i l_i^4}{EI_i}\frac{\partial^2 w_i}{\partial t^2} = 0 \;\; (\imath = 1, 2, ..., 6), \qquad (1)$$

where E is the elasticity modulus of the boom material; I_i and μi are averaged axial moment of inertia of the cross section and specific mass of the link section; $\xi_i = x_i / l_i$ is the relative longitudinal coordinate; t is time.

The rotation angle of the cross section of this link, the bending moment, and the transverse force are defined as

$$\varphi_i = \frac{1}{l}\frac{\partial w_i}{\partial \xi_i}; \quad M_i = -\frac{EI_i}{l_i^2}\frac{\partial^2 w_i}{\partial \xi_i^2}; \quad Q_i = -\frac{EI_i}{l_i^3}\frac{\partial^3 w_i}{\partial \xi_i^3}$$

$$(i = 1,\ 2,\ \ldots,\ 6). \qquad (2)$$

Boundary conditions for the hinged end of the lower section are written in the form

$$w_1(0,t) = 0; \quad M_1(0,t) = 0. \qquad (3)$$

The conditions of conjugation of sections belonging to one link are expressed by relations

$$w_{i+1}(0,t) = w_i(1,t); \quad \varphi_{i+1}(0,t) = \varphi_i(1,t)$$
$$(\iota = 1, 2, 4, 5). \qquad (4)$$

Conditions of conjugation of hinged links, one of which belongs to the lower and other to the upper link, are written, according to the scheme shown in Fig. 2, *a*, in the form

$$w_4(0,t) = -w_3(1,t)\cos\alpha\,;$$
$$\varphi_4(0,t) = \varphi_3(1,t) - \frac{1}{c_\varphi} M_3(1,t), \qquad (5)$$

where $\alpha=\alpha_1+\alpha_2$ is the angle between the axes of the sections of the mechanical system.

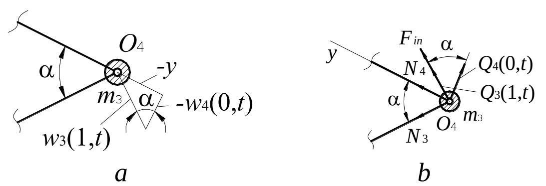

Fig. 2. Diagram of (*a*) the displacements of the hinge by which the robot links are joined and (*b*) the forces acting on the hinge

Conditions of equality of bending moments on common borders of the adjacent sections are given as

$$M_{i+1}(0,t) = M_i(1,t) \qquad (\iota = 1, 2, \ldots, 5). \qquad (6)$$

For the joints of the link sections, the transverse forces satisfy the conditions

$$Q_2(0,t) = Q_1(1,t) + m_1\frac{\partial^2 w_1(1,t)}{\partial t^2} + c_w w_1(1,t); \qquad (7)$$

$$Q_3(0,t) = Q_2(1,t) + m_2\frac{\partial^2 w_2(1,t)}{\partial t^2}; \qquad (8)$$

$$Q_{i+1}(0,t) = Q_i(1,t) \ \ (\iota = 4, 5). \qquad (9)$$

The hinge by means of which links of the mechanical system are joined, undergoes the action of forces Q_3(1, t) and N_3 from the lower part of a structure (Fig. 2, b) and forces Q_4(0, t) and N_4 from the upper part. In addition, the hinge is under the action of the inertia force F_{in} = m_3(д$_2$$w_3$(1,t))/дt$_2$ of the component with a concentrated mass m_3. According to the scheme of displacements of the hinge center (Fig. 2, a) and the scheme of forces acting on the hinge (Fig. 2, b), we write the following relations:

$$y = -w_3(1,t)\sin\alpha; \qquad (10)$$

$$m_\Sigma \frac{d^2 y}{dt^2} + N_4 = 0; \qquad (11)$$

$$Q_3(1,t) + m_3\frac{\partial^2 w_3(1,t)}{\partial t^2} + Q_4(0,t)\cos\alpha + N_4\sin\alpha = 0, \qquad (12)$$

where m_Σ is the total mass of the upper system.

Differentiating expression (10) twice over time and excluding the unknown functions y and N_4 from equations (10)–(12), we obtain

$$Q_4(0,t) = -\left(\frac{m_3}{\cos\alpha} + \frac{m_\Sigma \sin^2\alpha}{\cos\alpha}\right)\frac{\partial^2 w_3(1,t)}{\partial t^2} - \frac{1}{\cos\alpha}Q_3(1,t). \qquad (13)$$

The boundary conditions for the free end of the upper link are in the form

$$M_6(1,t) = 0; \qquad Q_6(1,t) + m_4\frac{\partial w_6(1,t)}{\partial t^2} = 0. \qquad (14)$$

Thus, the analysis of free oscillations of the considered mechanical system of the robot is reduced to finding such solutions of equations with partial derivatives (1), which would satisfy the boundary conditions (3)–(9), (13), and (14).

In the case of harmonic oscillations of the mechanical system, the solution of equations (1) is found in the form

$$w_i(\xi,t) = W_i(\xi_i)\sin\omega t \qquad (\iota = 1, 2, \ldots, 6), \qquad (15)$$

where $W_i(\xi_i)$ are amplitude functions of flexures of link sections.

The rotation angle of the cross section and the internal force factors of the link section, taking into account (2) and (15) are given as

$$\varphi_i(\xi,t) = \Phi_i(\xi_i)\sin\omega t\,; \ M_i(\xi,t) = M_i^*(\xi_i)\sin\omega t\,;$$
$$Q_i(\xi,t) = Q_i^*(\xi_i)\sin\omega t \qquad (\iota = 1, 2, \ldots, 6), \qquad (16)$$

where $\Phi_i(\xi_i)$, $M^*_i(\xi_i)$, $Q^*_i(\xi_i)$ are amplitude functions of rotational displacement of the cross section and the corresponding internal force factors.

Dividing the variables in equations (1) taking into account (15), we obtain differential equations of amplitude functions

$$\frac{d^4 W_i}{d\xi_i^4} - c_i^4 W_i = 0 \qquad (\iota = 1, 2, \ldots, 6), \qquad (17)$$

where

$$c_i^4 = \frac{\omega^2 \mu_i l_i^4}{EI_i}.$$

According to the method of initial parameters [7, 8], the solutions of equations (17) are given in matrix form

$$Y_i(\xi_i) = S_i(\xi_i)Y_i(0) \qquad (\iota = 1, 2, \ldots, 6), \qquad (18)$$

where

$$Y_i(\xi_i) = \mathrm{col}\big(W_i(\xi_i),\ \ W_i'(\xi_i),\ \ W_i''(\xi_i),\ \ W_i'''(\xi_i)\big),$$

$$S_i(\xi_i) = \begin{pmatrix} \psi_{1i}(\xi_i) & \psi_{2i}(\xi_i) & \psi_{3i}(\xi_i) & \psi_{4i}(\xi_i) \\ \psi_{1i}'(\xi_i) & \psi_{2i}'(\xi_i) & \psi_{3i}'(\xi_i) & \psi_{4i}'(\xi_i) \\ \psi_{1i}''(\xi_i) & \psi_{2i}''(\xi_i) & \psi_{3i}''(\xi_i) & \psi_{4i}''(\xi_i) \\ \psi_{1i}'''(\xi_i) & \psi_{2i}'''(\xi_i) & \psi_{3i}'''(\xi_i) & \psi_{4i}'''(\xi_i) \end{pmatrix}. \qquad (19)$$

Here is the fundamental system of integrals of equation (17), determined by formulas

$$\psi_{1i}\left(\xi_i\right)=\frac{1}{2}\left(\operatorname{ch} c_i\xi_i+\cos c_i\xi_i\right),$$

$$\psi_{2i}\left(\xi_i\right)=\frac{1}{2}\left(\operatorname{sh} c_i\xi_i+\sin c_i\xi_i\right),$$

$$\psi_{3i}\left(\xi_i\right)=\frac{1}{2}\left(\operatorname{ch} c_i\xi_i-\cos c_i\xi_i\right),$$

$$\psi_{4i}\left(\xi_i\right)=\frac{1}{2}\left(\operatorname{sh} c_i\xi_i-\sin c_i\xi_i\right). \qquad (20)$$

Considering relations (2), (15), and (16) together, we obtain matrix equations

$$F_i\left(\xi_i\right)=B_{1i}Y_i\left(\xi_i\right), \quad Y_i\left(\xi_i\right)=B_{2i}F_i\left(\xi_i\right) \quad (\iota=1, 2, ..., 6), \qquad (21)$$

where

$$B_{1i}=\operatorname{diag}\left(1,\ 1/l_i,\ -EI_i/l_i^2,\ -EI_i/l_i^3\right), \qquad (22)$$

$$B_{2i}=\operatorname{diag}\left(1,\ l_i,\ -l_i^2/\left(EI_i\right),\ -l_i^3/\left(EI_i\right)\right). \qquad (23)$$

From relations (18) and (21) it follows that

$$F_i\left(\xi_i\right)=B_{1i}S_i\left(\xi_i\right)B_{2i}F_i\left(0\right) \quad (\iota=1, 2, ..., 6). \qquad (24)$$

Taking into account (15) and (16) we exclude from the boundary conditions (4)–(9), (13), (14) the functions of time and write them in the form

$$F_{i+1}\left(0\right)=R_iF_i\left(1\right) \ (\iota=1,2,...,5); \quad F_7=R_6F_6\left(1\right), \qquad (25)$$

where

$$R_1=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\c_w-m_1\omega^2&0&0&1\end{pmatrix};$$

$$R_2=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-m_2\omega^2&0&0&1\end{pmatrix};$$

$$R_3=\begin{pmatrix}-\cos\alpha&0&0&0\\0&1&-1/c_\phi&0\\0&0&1&0\\\left(m_3+m_\Sigma\sin^2\alpha\right)\omega^2/\cos\alpha&0&0&-1/\cos\alpha\end{pmatrix}$$

$$R_i=\operatorname{diag}\left(1,\ 1,\ 1,\ 1\right) \quad (i=4, 5);$$

$$R_6=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-m_4\omega^2&0&0&1\end{pmatrix} \qquad (26)$$

moreover,

$$F_7=\operatorname{col}\left(W_6\left(1\right),\ W_6\left(1\right),\ 0,\ 0\right). \qquad (27)$$

According to the method of initial parameters [7, 8], the solutions of equations (17) are given in matrix form

$$W_1\left(0\right)=0; \quad M_1^*\left(0\right)=0. \qquad (28)$$

Taking into account the dependences (24)–(28), we write the matrix relation

$$F_7=\prod_{i=6}^{1}\left(R_iB_{1i}S_i\left(1\right)B_{2i}\right)F_1\left(0\right), \qquad (29)$$

where

$$F_1\left(0\right)=\operatorname{col}\left(0,\ \Phi_1\left(1\right),\ 0,\ Q_1^*\left(1\right)\right). \qquad (30)$$

The reactions of the third and fourth elements of the matrix-column F7 expressed by relation (27), to the unit values of the second and fourth elements of the matrix-column F1, which has the form (30), are denoted by m_φ, m_q, q_φ, and q_q, respectively. In order for the third and fourth elements of the matrix-column F7 to be zero, the following relations must be satisfied

$$m_\varphi\Phi_1\left(1\right)+m_qQ_1\left(1\right)=0;$$

$$q_\varphi\Phi_1\left(1\right)+q_qQ_1\left(1\right)=0. \qquad (31)$$

According to the method of initial parameters [7, 8], the solutions of equations (17) are given in matrix form

$$m_\varphi q_q-m_qq_\varphi=0. \qquad (32)$$

The values of m_φ, m_q, q_φ, and q_q, included in equations (32), are functions of the cyclic frequency ω and are determined by the matrix equation (29). From condition (32) we determine the natural frequencies of the mechanical system of the robot. Forms of oscillations of link sections are found by dependence (18) taking into account relations (19) and (20). In order to form a matrix-column of the initial parameters of the first section (30) up to a constant multiplier, we determine the unknowns of a homogeneous system of equations (31). The initial parameters of other sections are found by the formula

$$F_i\left(0\right)=\prod_{j=i-1}^{1}\left(R_jB_{1j}S_j\left(1\right)B_{2j}\right)F_1\left(0\right) \quad (i=2, 3, ..., 6),$$

which results from relations (24) and (25).

3. RESULTS OF MODAL ANALYSIS AND CONCLUSIONS

The considered method of calculation of frequencies and forms of free oscillations of a two-link mechanical system of a robot manipulator can be used in systems of the automated designing of the technological equipment. The application of the method of initial parameters promotes the systematization of the computational process and facilitation of the numerical implementation of the method using a computer.

For example, let us define the frequencies of free oscillations of a mechanical system of the robot, which has the following parameters: l_1 = 2.10 m; l_2 = 3,55 m; l_3 = 2.96 m; $l_4 = l_5 = l_6$ = 3.70 m; $A_1 = A_2 = A_3$ = 0.7812·10^{-2} m; A_4 = 0.7216·10^{-2} m; A_5 = 0.6312·10^{-2} m; A_6 = 0.5408·10^{-2} m; $I_1 = I_2 = I_3$ = 0.1792·10^{-3} m^4; I_4 = 0.2016·10^{-3} m^4; I_5 = 0.1607·10^{-3} m^4; I_6 = 0.7137·10^{-4} m^4; m_1 = 42.30 kg; m_2 = 42.40 kg; m_3 = 54.70 kg; m_4 = 110.5 ... 210.5 kg; c_w = 0.1270·10^7 N/m; c_φ = 0.8730·10^7 N·m/rad; α = 0.5 ... 3.0 rad.

The results presented in the table show that the natural frequencies of the mechanical system of the robot significantly depend not only on the elastic-inertial characteristics of its components, but also on the relative position of the links. Therefore, the modal analysis of mechanical systems of this type is essential both for creation of rational designs of robots, and for maintenance of favorable modes of their operation.

Table 1. The natural frequencies of the two-section boom of the cranked lifting device

m4, kg	α, rad	Frequency values, Hz				
		1	2	3	4	5
210,5	0.5	1.250	1.739	12.78	19.27	38.89
	1.0	1.009	1.752	11.97	18.50	37.76
	1.5	0.8110	1.973	11.67	18.46	37.53
	2.0	0.6916	2.390	11.70	18.91	37.76
	2.5	0.6270	3.030	11.99	20.28	38.77
	3.0	0.6013	3.610	12.36	22.63	40.99
110,5	0.5	1.426	1.785	13.72	19.27	39.96
	1.0	1.104	1.868	12.87	18.53	38.83
	1.5	0.8844	2.108	12.54	18.52	38.59
	2.0	0.7605	2.536	12.53	19.01	38.80
	2.5	0.6956	3.204	12.75	20.48	39.76
	3.0	0.6702	3.823	13.02	23.00	41.83

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