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USING THE INSTANTANEOUS ANGULAR SPEED MEASUREMENT TO CHARACTERIZE THE TRANSIENT DYNAMIC RESPONSE OF AN INERTIAL SYSTEM

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Abstract

The measurement of instantaneous angular speed in flywheels has been used in Internal Combustion Engines for the diagnosis of faults in the fuel injection systems, combustion quality at idle, ignition system, and especially in the performance assessment of starter motor and battery accumulator, among others.

It is the aim of this paper to forecast the use of an experimental flywheel-based test bench, driven by a single-cylinder Internal Combustion Engine and a starter motor, developed to perform research studies in the laboratory of internal combustion engines. It is illustrated the measurement and processing of the instantaneous angular speed of a flywheel to characterize the dynamic response of the inertial system during the run-up and run-out regimes. The mathematical model of the transfer function is presented, relating the angular velocity with the torque transmitted by the starter motor as a first-order system. For the acquisition of the signal, a NI 6009 card is used, while a Matlab computer program is employed to plot the instantaneous angular velocity curves, and also to identify the time response of the system. The time constant is 1.54 seconds, which corresponds to 63.2% of the value of steady state signal in run-up regime.

Keywords: dynamic response, transient state, starter motor, instantaneous angular velocity, inertial system

1. INTRODUCTION

The measurement of instantaneous angular speed in flywheels has been approached in different research works for the diagnosis of faults (problems in the fuel injection system, evaluation of engine combustion quality, problems in the starter motor and batteries) in internal combustion engines. The process of measuring the instantaneous angular speed in flywheels can not only be useful in the fault diagnosis of Internal Combustion Engines, but also constitutes the basis for measuring their rotational kinetic energy, since this type of storable energy is a function of the moment of inertia and the angular velocity.

A. Charchalis and M. Dereszewski [1] employ the instantaneous angular speed technique for the diagnosis of fuel injection system failure (fuel injection system failures deteriorate the quality of combustion and reduce fuel efficiency, engine performance) in a 4-stroke, three-cylinder diesel engine that drives a three-phase generator. For the measurement of instantaneous angular speed, they used an Optical Encoder, an ETNP-10 counting system, and through a polynomial approximation (the Savitzky-Golay filter), they optimized the measured signal. The authors obtained curves of angular velocity as a function of crankshaft rotation angle for healthy and faulty conditions (fuel injection system failure). For the engine under study, they built a dynamic model and calculated standard deviations in the measured data from which they derived useful information. The difference in relative speeds of the curves obtained provided good information regarding the diagnosis of the engine condition. On the other hand, Lang K, Liu L and Lang A [2] use a multipurpose flywheel on which angular velocity measurements are used to diagnose misfire. The multipurpose flywheel has one of its teeth made of a non-ferrous material, which is used as a reference mark. The authors in the experimentation employ Hall effect sensors and variable reluctance sensor for measuring the angular position of the flywheel. The variable reluctance and hall effect sensors are mathematically modeled in equations 1 and 2, respectively:

$$e = -N \cdot \frac{d\Phi}{dt} \qquad (1)$$

$$u = \frac{R \cdot I \cdot B}{d} \qquad (2)$$

In the variable reluctance sensor, an electromotive force e is generated which depends on the number of turns in the coil, N and the rate of change of the magnetic flux $\frac{d\Phi}{dt}$. The Hall effect element in the sensor generates a Hall voltage u which is proportional to the flux density of the magnetic field B which is perpendicular to the flow of the supply current I, R is the Hall constant and d is the thickness of the conductor. The non-ferrous flywheel tooth is invisible to magnetic sensor pick-up and serves as a reference mark on the measured signal. The authors obtained signals for voltage versus time and angular velocity versus revolutions for sensors placed on the front and rear of the engine.

Blair G and Kee Robert [3] describe the use of an inertial dynamometer, which incorporates a flywheel, as an experimental test system for measuring power under acceleration conditions. An optical encoder is coupled to the flywheel of the dynamometer to provide the frequency signal, which is converted to an analog voltage by an integrated circuit; this voltage is proportional to the speed of the flywheel and is connected to an input of the data acquisition system. A second input channel is used to record the temperature. In evaluating the performance of the 100 cm^3 Kart engine, they used two exhaust systems with different characteristics and curves of power versus angular speed and effective mean pressure versus angular speed were obtained in a range of 6250 min^{-1} up to 16000 min^{-1}. Acceleration characteristics are presented in starting systems for its importance in transport systems [9].

Henry R, Lequesne B and Chen S studied the dynamic response of an alternator-starter system driven from rest to a stable state; they obtained the evolution of the angular velocity curves as a function of the angle of rotation, the torque on the motor shaft, and the frictional torque. Finally, a mathematical model is obtained [10].

In correspondence with the previous article, the characteristic curve of the angular velocity of the crankshaft as a function of time is shown, with the starter motor actuating from a stationary value until the system reaches a steady-state value [11].

Procedures for instantaneous angular velocity measurement for monitoring and control purposes in rotating machinery are reviewed. Instantaneous angular velocity measurement has become important because it allows establishing the dynamic response of a machine [12]. A review of the main instantaneous angular speed measurement techniques and processing techniques used in various fields of study is also presented, as well as the errors associated with its measurement, where imperfections are the cause of geometric errors [15][16].

Wojnar and Lazarz present methods for estimating time-delay in the process of vibration signals' synchronous averaging obtained in rotating machinery, especially damage presented in gear wheels. Synchronous averaging is applied to improve the signal-to-noise ratio and suppress unwanted signals that are not related to the period of rotation. Different vibration signals are presented depending on the angle of rotation, where a synchronously averaged signal allows its analysis in a simpler way in terms of detecting the failure in the gear wheel [13]. Also, the use of the measurement of torsional vibrations in toothed shafts for the detection of faults is complemented, the signals here are processed through the use of the wavelet transform [14].

In internal combustion engines, it is important to recognize the quality of combustion; as this will affect the performance of the engine. Teng C [4] uses the flywheel acceleration method to evaluate the quality of combustion. In experimentation he uses a V8 engine, deriving the angular acceleration curve as a function of the crankshaft angular position. The RMS value of the angular acceleration is determined and curves of RMS_{α} against angular position are constructed for 300 cycles of the engine. From these curves the engine combustion stability is evaluated, correlating the values RMS_{α} and IMEP (indicated mean effective pressure).

The measurement of instantaneous angular speed data in flywheels for the reconstruction of the internal pressure in the cylinders requires the use of physical equations that govern the behavior of the system (combustion and dynamics). Authors such as Peragón F, Jímenez Francisco and Aguilar E [5] use neural networks to simplify the complexity of the problem of reconstructing the internal pressure in the cylinders from values of angular velocity; circumventing the use of dynamic relationships for the moving parts, as well as the combustion process models.

To obtain the engine performance curves, it is required to measure the torque and the mechanical power in the crankshaft axis. An alternative method is to resort to the free acceleration technique, which is based on measurements of angular velocity on flywheels, to obtain the operating curves of the engine; this is dealt with by Merchan C [6].

Regarding the applications of regenerative braking systems, Hsu Tai-Ran [7] and his team developed an experimental test bench to measure the kinetic energy stored in the flywheel during braking. Angular velocity curves as a function of time were obtained and from the kinetic energy equation, the energy stored in a time interval was estimated.

This paper addresses the process of measuring the instantaneous angular speed to characterize the transient dynamic response of an inertial system. In the following parts of this work it will be described the specially developed flywheel inertial test bench with its instrumentation, its use, and the methodology followed to perform the identification procedure to obtain the mathematical model of the inertial system.

2. CHARACTERISTICS OF THE EXPERIMENTAL TEST BENCH

Following, the elements that constitute the test bench, dimensioned and selected according to the design process are presented:

- internal combustion engine: A single cylinder 7,35 kW internal combustion engine was available in the internal combustion engine laboratory.
- battery: The battery or accumulator is intended to power the starter motor; in this case, 12 volts are required, 75 A-h battery is used.
- starter motor: A conventional starter motor was available, the main component to be observed in the investigation. The technical characteristics of the starter motor are presented in table 1.

Table 1. Technical characteristics of the starter motor

Application	Gasoline Engines up to 1,6 liter displacement.
Tension	12 Volts
Power	1 KW
Locking Torque	10,5Nm (1,07Kgm)
Rotation	Clockwise or counteclockwise acording to requiretments of operation
Temperature	-20° C a 110°C (-4°F a 230° F)
Weight	3 kg
Useful Life	More than 60,000 maintenance-free starts. (10 years minimum of use)
Mounting	Designed exactly for every application

The electric starter motor used in the experimental test bench is triggered by a pushbutton, generating a torque when meshing with the flywheel. A double channel pulley at the end of the shaft is foreseen for the test cases when the internal combustion is to be used. During the starting process, the starter motor must overcome the resistances of flywheel inertia and aerodynamic drag, mechanical losses and cylinder compression (in case of coupling the ICE through the belt drive). Figure 1 presents the starter motor used in the test bench, with its representation in Solidworks.

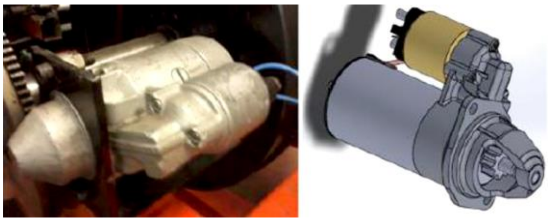

Fig. 1. Starter motor of the test bench with its representation in solidworks

Structure of the test bench: This structure is responsible for supporting the weight of each of the components that are powered by the internal combustion engine and the starter motor, and also supports the vibrations generated at the time of start-ups. The structure conceived in the design consists of a central base in the shape of a rectangular parallelepiped. At its ends, two structures were welded to support the weight of the combustion engine and the battery that is connected to the electric starter. These two structures are supported at the ends of the central base as shown in the diagram in figure 2.

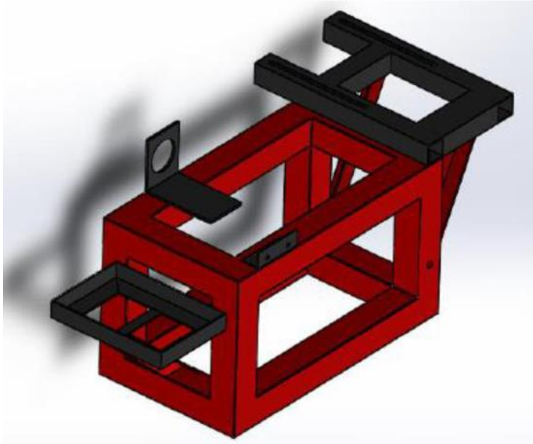

Fig. 2. 3D solid modeling of the metal structure made in Solidworks®

Rotor formed by flywheels: The starter motor drives the two flywheels aligned in their axis. Due to their own inertia they continue to rotate, even after turning off the starter motor; both simulate the inertia that the starter motor must run up, as shown in figure 3 of the assembly of the elements. On the outside diameter of the flywheels there is a toothed ring, which meshes with the starter motor pinion. The teeth of the sprocket are specially heat treated to guarantees a long service life.

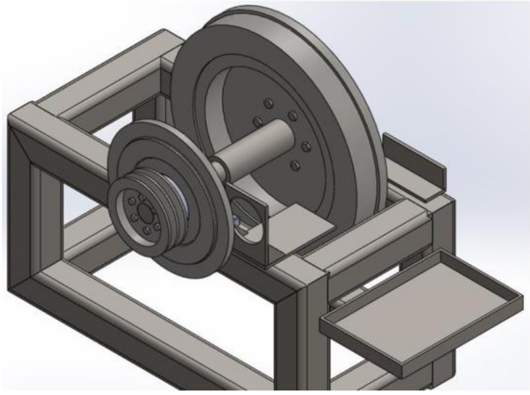

Fig. 3. Shaft assembly with flywheels made in Solidworks

Connecting box: It is in charge of powering the starter motor, it contains a starter button, a red-light bulb to indicate the starter operation, and an on/off switch, as shown in figure 4.

Fig. 4. Box to drive the starter motor

3. SENSOR SELECTION FOR ANGULAR POSITION MEASUREMENT

Among the most common sensors for angular velocity measurement are encoders, proximity sensors and photoelectric sensors. Tree encoders are the best solution for the device being sensed. They offer high resolution (typically 1 to 5000 pulse per revolution) and clearly defined symmetrical pulses. However, it is sometimes not feasible to mount an encoder on the shaft being monitored. Variable reluctance and Hall effect sensors are among the most popular proximity sensors used for detecting teeth in the automotive industry. These sensors are defined below along with their mathematical models that characterize them:

Variable Reluctance Sensor: A variable reluctance (VR) sensor typically consists of a coil and a permanent magnet core. The coil generates an electromotive force (emf), when the amount of magnetic flux that passes through the coil changes; The electromotive force, *e*, is based primarily on the rate of change of the magnetic flux, and the number of turns in the coil, *N*. This electromotive force is induced, as each ferromagnetic tooth passes the pickup; the reluctance of the magnetic circuit changes with time. Thus, the induced voltage will depend on the number of teeth and the angular velocity in its amplitude and frequency of the signal [8], this can be represented mathematically in equation 3, as:

$$E = C_B \cdot N_t \cdot \omega \cdot \sin N_t \omega t \qquad (3)$$

Where, E: Output voltage, C_B: Proportional constant, N_t: Number of teeth and ω: angular velocity of the wheel. The way of experimental measurement of the angular velocity on a gear wheel can be seen in figure 5.

Hall effect sensor: A Hall effect sensor works differently than a VR sensor. The Hall effect element in the sensor generates a Hall voltage that is proportional to the flux density of the magnetic field B perpendicular to the flow of the supply current I, as described by the equation 4:

$$u = \frac{R \cdot I \cdot B}{d} \qquad (4)$$

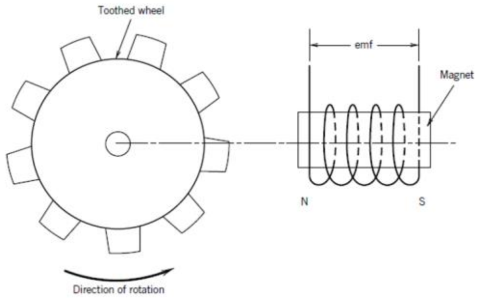

Fig. 5. Angular velocity measurement using a gear wheel and a magnetic pickup [8]

Where R is the Hall constant and d is the thickness of the conductor. With electronic circuitry internal to the sensor, a Hall effect sensor used for tooth detection typically generates high or low voltage levels based on the strength of the magnetic field. When a non-ferrous tooth gets close, it doesn't change the intensity of the magnetic field enough and the output stays the same.

As for photoelectric sensors, these are widely used for vehicle applications, since they replace mechanical devices with a beam of light that can be used at distances of less than 20 mm to several centimeters, according to the optical lenses used. They work with a light source that goes from the incandescent type, to the light-emitting diodes (LEDs), and they operate by detecting a change in the light received by the photo-detector.

The inductive sensor that was available in the laboratory of internal combustion machines presented good results in terms of the characterization that was carried out, in addition it has the advantage that it provides a digital signal. A CKP angular position sensor was also tested. This presented problems in terms of proximity and response on the multimeter. The encoders are a good option, but due to the construction conditions of the bench and taking advantage of the toothed ring of the flywheels, it was decided to use the inductive sensor. The experimental flywheel-based test bench instrumented together with the inductive sensor, a NI 6009 data acquisition card, the PC bus and a laptop, it can be seen in Figure 6.

4. DYNAMIC MODEL AND SYSTEM TRANSFER FUNCTION

The experimental test bench contains within its moving parts the belt drive system together with the flywheels and the power transmission shaft. The rotational system modeled by Newton's second law can be presented in the form in equation 5:

$$I_v \alpha + k_{rt} \theta = \tau_m(t) - \tau_{fr}(t) \qquad (5)$$

Where I_v is the total moment of inertia of the system, $\alpha = \frac{d\omega}{dt}$ corresponds to the angular acceleration experienced by the inertial system, k_{rt} is the torsional stiffness constant, $\tau_m(t)$ torque transmitted by the drive motor and $\tau_{fr}(t)$ frictional torque (viscous friction, mechanical losses). Newton's second law equation can be written in terms of angular position θ as shown in Equation 6.

$$I_v \frac{d^2\theta}{d^2t} + k_{rt}\theta = \tau_m(t) - \beta\frac{d\theta}{dt} \quad (6)$$

Fig. 6. Instrumented experimental test bench.

The instantaneous angular speed measurement in the experimental test bench will be composed of the blocks shown in figure 7.

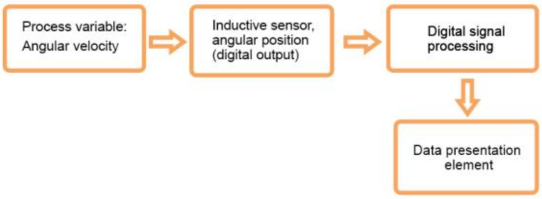

Fig. 7. Block diagram of the angular velocity measurement system.

I_v states for the moment of inertia of the whole system, k_{rt} is the torsional stiffness constant, θ is the angular position, β is the loss constant, $\tau_m(t)$ the transmitted torque. This model in its form, constitutes a second order model. Representing the previous differential equation in the frequency domain, applying the Laplace transformation with null initial conditions, it is obtained the equation 7:

$$G(s) = \frac{\Theta(s)}{T_m(s)} = \frac{1}{Is^2 + \beta s + k_{rt}} \quad (7)$$

The resulting transfer function relates the angular position to the transmitted torque. Now if it is written the differential equation for the system in terms of angular velocity, not including the torsional stiffness constant, to simplify the model a bit, the expressions shown in equations 8 and 9 respectively are obtained:

$$I_v \frac{d\omega}{dt} = \tau_m(t) - \tau_{fr}(t) \quad (8)$$

$$I_v \frac{d\omega}{dt} = \tau_m(t) - \beta\omega(t) \quad (9)$$

Where I_v is the total moment of inertia of the system, ω(t) is the angular velocity in time, $\tau_m(t)$ torque transmitted by the drive motor and τ_{fr} frictional torque (viscous friction, mechanical losses) in the supports of the shaft (bearings) which can be approximated as $\beta\omega(t)$. Applying the Laplace transform on both sides of the equation can be written as follows:

$$I_v s\,\Omega(s) = T_m(s) - \beta\Omega(s) \quad (10)$$

The transfer function of the system that relates the angular velocity to the transmitted torque can be written as follows:

$$G(s) = \frac{\Omega(s)}{T_m(s)} = \frac{1}{I_v s + \beta} \quad (11)$$

As can be observed in equation 11, this corresponds to the model of a first order system, whose time constant in this case is the quotient between the moment of inertia and the global loss constant. In the case of the variables to be measured on the test bench, the model proposed in equation 11 is of more interest, since the main variable to be recorded is the instantaneous angular speed at the flywheel.

5. METHODOLOGY OF EXPERIMENTAL TESTS

The methodology for these tests consists of operating the experimental test bench by means of the internal combustion engine at different speed regimes. The measured angular position signal is taken to a computer by means of the data acquisition system (DAQ) and by means of the Matlab program, the signal acquisition and processing routines are available.

The tests carried out include the following steps:

– the ICE is decompressed and the switch is turned ON to allow current to flow between the battery and the starter solenoid.
– the push button on the test stand ignition box is pressed to operate the starter motor.
– once the ICE has been started, the desired angular speed is adjusted by means of the accelerator (the ICE during the tests was set at 4 different angular speeds).
– the SKF model TMR 1 digital tachometer is positioned on the pulley attached to the flywheel assembly to record angular velocity. The corresponding data are taken for this speed regime.
– knowing the angular velocity at which the set of flywheels is rotating, press Run in the Matlab program to start the data acquisition.
– the previous steps are repeated to take the angular velocity data in the other operating regimes.

In the case of the drive with the starter motor, data acquisition was performed during the start time to explore the evolution of the angular velocity in this time period. It was used 10 seconds for the sampling time. For this, the following steps were carried out:

– the ICE is decompressed and the switch is turned ON to allow current to flow between the battery and the starter solenoid. The drive is carried out with the starter motor belonging to the ICE, which involves the transmission by gears between the starter motor and the flywheel of the

ICE and then the belt transmission towards the set of flywheels.
- the push button on the test stand ignition box is pressed to operate the starter motor.
- at the moment of start-up, press Run in the Matlab program to begin with the data acquisition.

6. RESULTS AND DISCUSSION

A Fluke brand oscilloscope was used, the variation of the signal over time was observed, it can be seen in figure 8.

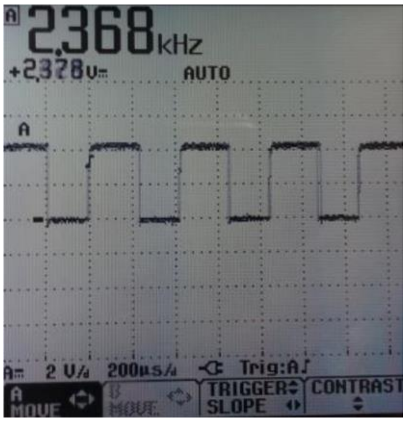

Fig. 8. Electrical voltage signal delivered by the inductive sensor in the oscilloscope

The value of angular velocity in time according to the signal of figure 8 can be determined by means of the following equation, where the frequency of the signal f_s and the number of teeth of the flywheel N_t are related.

$$n\,[\text{min}^{-1}] = \frac{f_s \cdot 60}{N_t} \qquad (12)$$

Replacing the frequency value obtained from the signal, and bearing in mind that the number of teeth of the flywheel of the experimental test bench is equal to 149 teeth, the following value of angular velocity is obtained:

$$n\,[\text{min}^{-1}] = \frac{f_s \cdot 60}{N_t} = \frac{2368\text{ Hz} \cdot 60}{149} = 953.56\text{ min}^{-1}$$

A Matlab code is written to obtain the values of angular velocity in time from the sampled signal. The figure 9 presents a block diagram of the algorithm used to determine the instantaneous angular velocity curves.

A code was also developed to apply a digital filter to the angular velocity versus time sampled signal in order to reduce noise presented in the signal. Thanks to the developed code, the curves were plotted for the data stored along different angular velocities. The red signal corresponds to an unfiltered signal and the blue signal is associated with the filtered signal. A part of the lines of code implemented in the programming can be seen in figure 10.

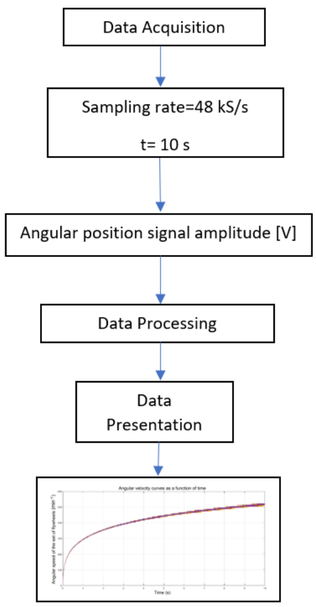

Fig. 9. Flowchart for the algorithm used with the angular position signal

```
a = 1;
b = 0;
data1 = zeros(size(data));
velocidad = data1;
t0=0;i=1;
for k=1:length(data)
    if(data(k)>=a && b==0)
        b=1;
        data1(k) = 1;
        t1 = time(k);
        if(t0==0)
            time1(i) = time(k);
        else
            dt = t1-t0;
            time1(i)=time(k);
            velocidad1(i) = 1/dt*60/149;
        end
        i = i+1;
        t0 = t1;
    end
    if(data(k)<=a)
        b = 0;
    end
end
```

Fig. 10. Lines of code for determining the instantaneous angular speed.

Figure 11 shows the angular velocity of the large flywheel of the test bench when driven by the internal combustion engine for a constant speed.

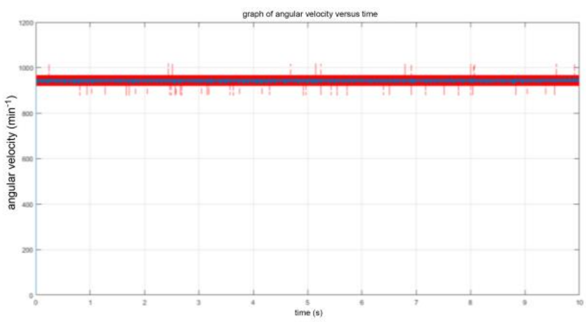

Fig. 11. Angular velocity as a function of time for a constant speed of 942 min^{-1}

According to the steps described to perform the angular velocity tests at different operating regimes, an SKF brand digital tachometer model TMRT-1 was used which is shown in figure 12.

Fig. 12. SKF brand Digital Tachometer

The angular velocity values obtained from the digital tachometer and the sampled signal are presented in table 2, where the relative error calculated for these values are determined.

Table 2. Angular velocity values obtained from the sample signal and the skf digital tachometer

Average angular velocity measured with digital tachometer [min^-1]	Average angular velocity obtained from the sampled signal [min^-1]	Relative error	Percentage error (%)
915,6	942,2	0,0291	2,91
1288	1288,5	0,0004	0,04
1401	1411,9	0,0078	0,78
1763,7	1807,2	0,0247	2,47

In accordance with the data obtained from table 2, comparing the angular velocity measured with the digital tachometer and the angular velocity obtained from the sampled signal, the percentage error values are not greater than 3% for speed regimes that were adjusted by means of the throttle on the internal combustion engine. The errors in the measurement of the angular velocity depend on the number of pulses obtained in one revolution (corresponding to the number of teeth on the flywheel) and the value of the period of the sampled signal t_{s2}, that is, between the consecutive peaks of the square signal of the figure 8. This value of time between consecutive peaks of the signal is what induces the systematic measurement error $\varepsilon_t = \pm t_{s2}$, so its reduction is achieved by increasing the number of pulses per revolution, that is, having a gear with a greater number of teeth and this in turn will allow a greater angular resolution to be achieved. The methods for estimating time- delay in the process of vibration signals' synchronous averaging are very useful in rotating elements such as gear wheels for estimating their instantaneous angular velocity and reducing noise in the measured signal that is not associated with the period of rotation [13], these methods are undoubtedly related to the measurement of the instantaneous angular speed signal carried out in these experiments and the synchronous averaging process has an impact on the relative errors found in Table 2. Also, the angular resolution $\Delta\theta$ at the flywheel ring, can be determined by means of the following equation:

$$\Delta\theta = \frac{360°}{N_t} = \frac{360°}{149} = 2.4° \approx 0.042 \text{ rad}$$

Figure 13 presents the curve of angular velocity as a function of time during startup. It can be observed in the behavior of the speed curve, that from the moment the starter began to overcome the inertia of the flywheel, it progressively increased its speed until reaching a point at which it transmitted a speed of approximately 350 min^{-1} to the flywheel axis.

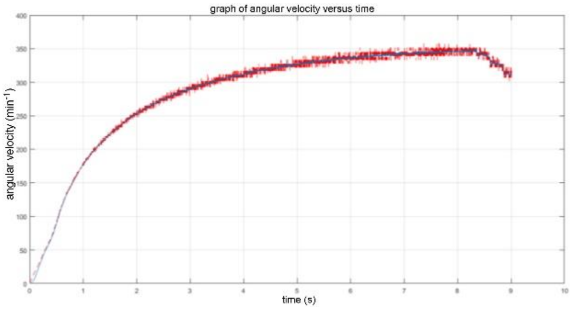

Fig. 13. Angular velocity versus time curve during system startup.

In this way, the evolution of revolutions over time can be adjusted to the transient response curve of a first-order system. For this, a code was written in Matlab, which allows generating a curve approximate to the real behavior.

The model of the instantaneous angular velocity curve can be written in the form:

$$\omega(t) = 350 \cdot \left(1 - e^{-\frac{t}{\tau}}\right) \quad (13)$$

Figure 14 shows the real curve constructed from the data acquired in blue for the filtered signal, red for the unfiltered signal, and in green the fit curve at these values.

In the case of the previous model obtained, the constant 350 corresponds to the steady state value of the revolutions in time, τ is the time constant, which, in correspondence with the mathematical model of the system, is determined as the quotient between the total inertia moment I_v and the loss constant β, which encompasses mechanical losses, viscous friction in the bearings and aerodynamic friction with the surrounding air. If the analogy is made with an electrical circuit, the capacitance of the capacitor is associated with the inertial value of the flywheel, while the resistance R is related to the loss constant. The value of the inertia of the set of flywheels was obtained from the Solidworks computer program. Figure 15 shows the solid modeling of the set of flywheels, from which the inertial value was estimated (Lzz = 2.55 kg.m^2 computer program), $I_v = 2.55 \text{ kg} \cdot m^2$. The value of β after performing some iterations, a suitable value of 1.65 $\frac{kg.m^2}{s}$ was found.

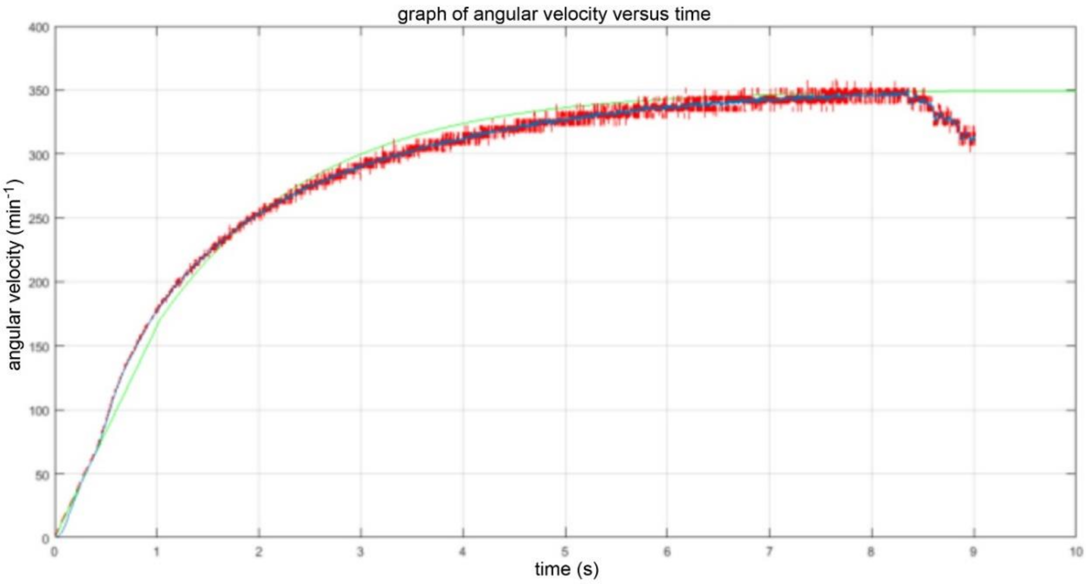

Fig. 14. Adjustment of the angular velocity curve in time.

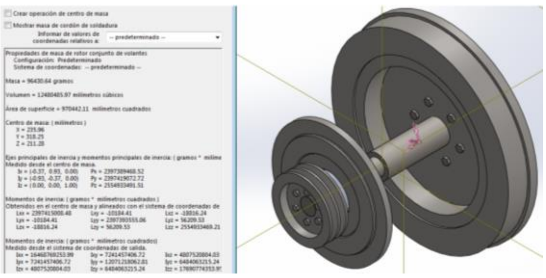

Fig. 15. Determination of the inertial value using the solidworks computational tool

The time constant for the curve of the transient response of the system is equal to 1.54 seconds. The angular velocity value for which 63.2% of the steady-state signal is reached is 221 min^{-1}. The mathematical model of the system can be rewritten as follows:

$$G(s) = \frac{\frac{1}{\tau}}{s + \frac{1}{\tau}} = \frac{\frac{1}{1.54}}{s + \frac{1}{1.54}} \qquad (14)$$

The curves obtained for angular velocity as a function of time as can be seen in figure 16, show the evolution of the angular velocity during system startup. As previously expressed, this curve can be adjusted to the transient response of a first-order system whose time constant τ is related to the moment of inertia of the set of flywheels and the loss coefficient, also the behavior of this curve instantaneous angular velocity allows associating the energy stored during startup. The steady state value of the signal is reached around 350 min^{-1}. These curves of evolution of angular velocity during starting take into account the inertial resistance of the set of flywheels, the transmission of movement of the belt towards the coupled internal combustion engine and aerodynamic friction.

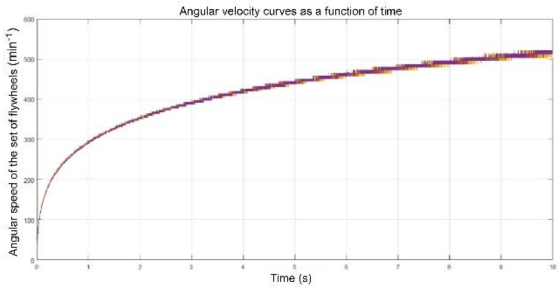

Fig. 16. Angular velocity signals over time during startup

In correspondence with the previous curves, and observing the behavior of the instantaneous angular speed during the energy delivery of the set of flywheels, figure 17 shows the combined curve for the moment in which the system accumulates and releases the rotational kinetic energy. In the curve it can be seen how the angular velocity evolves in time until it reaches a maximum value of 380 min^{-1} in 10 seconds, from that moment on, the angular velocity begins to decrease in time, as product of aerodynamic friction, bearing friction and internal combustion engine load through belt drive, to standstill in approximately 25 seconds.

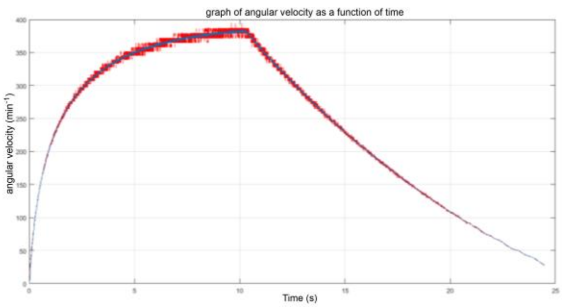

Fig. 17. Angular velocity curve over time during energy accumulation and delivery

As has been shown in the above drawings and pictures, the flywheel inertial test bench has been envisioned to drive machines as internal combustion engines, diverse type of motors, as well as different types of industrial rotary devices and machines and observe its dynamic response. Particularly, it has been designed also to facilitate the study of small internal combustion engines under driving and motored modes of operation, with compression and without compression. Figures 18 reveals the performance of the run-up of the engine driven by the external starter and also by its own starter motor.

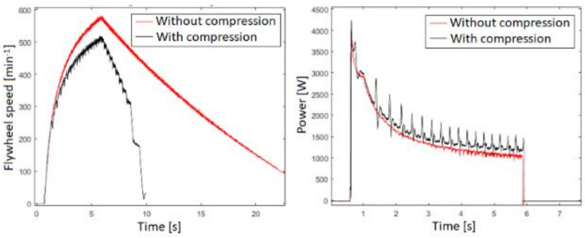

Fig. 18. Flywheel speed during run-up and run-out, and starter power consumed during the motor engine cranking

The engine and flywheel can be driven by the starter of the bench through the pulley-belt transmission, and also by the starter-motor of the combustion engine. In figure 18 there are represented the flywheel speed and power consumed by the engine starter while running up the flywheel-engine entire system, for the motored non-fired engine with fuel injector placed (there is compression inside the engine cylinder) and with fuel injector removed (there is no compression inside the engine cylinder).

During cranking, the instantaneous flywheel acceleration corresponding to each engine speed directly gives a measure of the torque and power characteristics. Setting-up properly an experiment, after characterizing the engine inertia, it is possible to diagnose the cranking performance under different conditions. Determining the mechanical losses in the system, and the aerodynamical drag, it will be possible to study the performance as well as the mechanical losses of the combustion engine. The mechanical losses along the subsystems of the test bench can be characterized by the free deceleration method.

7. CONCLUSIONS

It has been put into practice a method to measure the evolution of the angular velocity of a flywheel inertial system during its start-up, to associate it with the transient response of a first order system. The obtained transfer function for the set of flywheels of the experimental test bench relates the time change of angular velocity with the torque transmitted in the shaft that supports the set of flywheels. The time constant relates the inertial value of the set of flywheels $\mathrm{I} = 2.55\ \mathrm{kg.m^2}$ and a global loss constant with their values $\beta = 1.65\ \frac{kg.m^2}{s}$ (value associated with the friction losses in bearings, aerodynamic friction and friction in belt transmission) and $\beta = 0.9\ \frac{\mathrm{kg.m^2}}{\mathrm{s}}$ (value associated with friction in bearings and aerodynamic friction). The theoretical curve obtained compared well with the real curve of the angular velocity in time, during the actuation of the starter motor, presenting after the first 3 seconds relative errors below 0.2.

The quotient between the transmitted torque and the angular velocity can be established as a first order system. Effectively, the transient response corresponds with a first order system in the run up regime where the time constant is equal to 1.54s. and the speed for the steady state is 350 $\mathrm{min^{-1}}$.

The angular resolution given by the number of teeth of the flywheel crown was determined, whose value is equal to 2.4°, the angular resolution can be improved if the number of teeth used in the flywheel is increased, allowing it to increase the frequency in the signal and the temporal passage between consecutive peaks of the signal is reduced, achieving a greater quantity of velocity data with which to obtain the instantaneous angular velocity curve.

The internal combustion engine used to drive the flywheel assembly used a belt drive to the flywheel assembly. It is of interest to be able to establish the mechanical losses in the belt transmission due to the slip produced. Transmission error between pulleys, pulley/belt slip are deduced from pulley rotation angle measurements and it is necessary to be able to use optical encoders in the two pulleys of the test bench. The use of tensioner limits belt slip on pulleys, and can have a positive effect on error reduction. It should be clarified that these measurements are outside the scope of the experimental tests carried out on the test bench and it is not possible to view the influence of the slippage of the pulleys on the phase shift of the signals and on the error transmission.

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